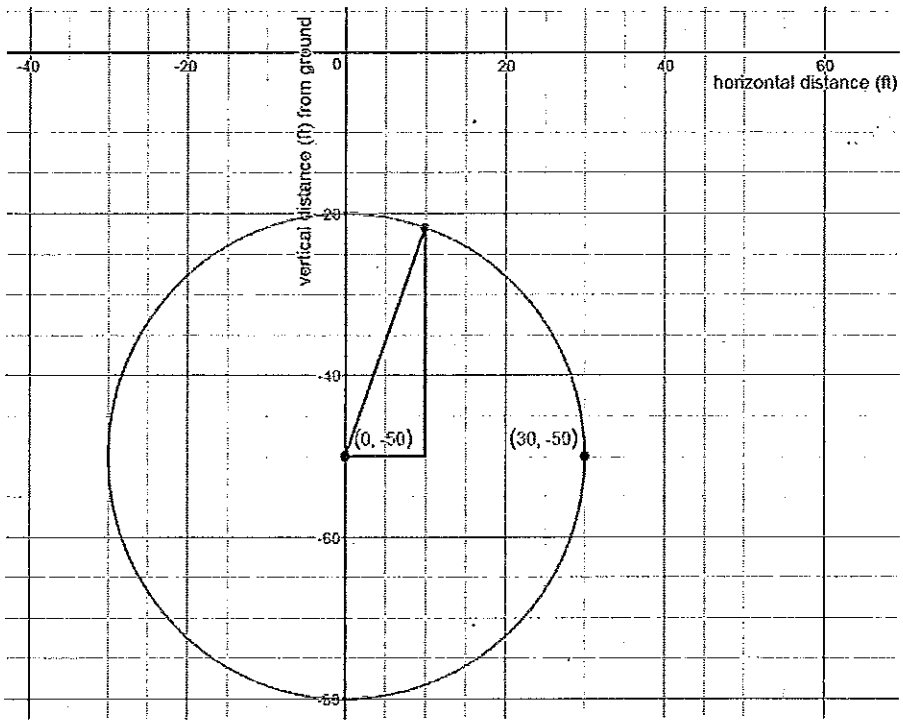
CCSS Algebra 4 Introduction to Trigonometric Functions

Seattle is currently building a tunnel beneath downtown to help ease traffic congestion. They are using Bertha, a giant tunneling machine seen here. Bertha cuts a circular hole with a radius of 30 feet. The engineers in charge of the project have to be aware of the depth of the tunnel as they are boring.

The image shows a cross section of the tunnel. In the image, x = the horizontal distance (in feet) from the center of the current road; y = the vertical distance from the ground (in feet). For example, the coordinate (30, -50) represents a point on the tunnel that is 30 feet to the right of the road center and 50 feet **below** the ground.



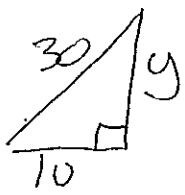
1. How far below the ground is the top of the tunnel? The very bottom of the tunnel?

Top = 20
Bottom = 80

2. The engineers are worried about water lines that are 10 feet to the right of the center of the road and 22 feet below the ground. Does it appear, based on the image above that Bertha will hit the water pipes? Explain why or why not.

Yes, (10, -22) looks like it is inside the circle.

3. Use the triangle drawn to determine for sure whether or not Bertha will hit the water line (hint: how long is the hypotenuse of the triangle? How long is the horizontal leg?).



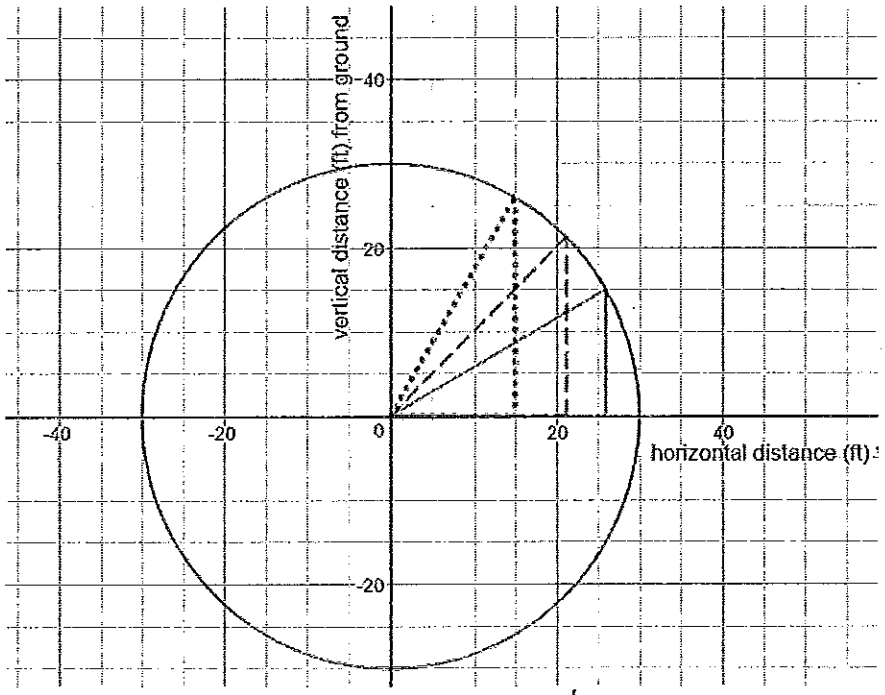
$10^2 + y^2 = 30^2$
$100 + y^2 = 900$
$y^2 = 800$
$y = \sqrt{800} = 28.28$

Use $a^2 + b^2 = c^2$ when you know sides, but not angles.

4. What is the equation of Bertha's circle?

$x^2 + (y+50)^2 = 30^2$

Near the entrance to the tunnel, the cross section of Bertha's tunneling looks like this:



5. What is different about the cross section shown here compared to the one on the previous page?

Origin is middle of Bertha's tunnel

6. What is the equation of Bertha's circle according to this picture?

$x^2+y^2=30^2$

7. **For each triangle shown (dotted, dashed, solid),**

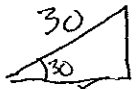
a. Determine the measure (in degrees) of the angle at the center of the circle.

30°, 45°, 60° (Info given in class)

Use sin when you know an angle & a length

b. Use this angle measure and the hypotenuse to determine the value of x and y for the points where the triangles intersect the circles.

Solid



$\sin 30 = \frac{y}{30}$

$30 \sin 30 = y = 15$

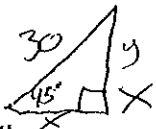
$x^2+15^2=30^2$

$x^2+225=900$

$x^2=675$

$x \approx 26$

Dashed



$\sin 45 = \frac{y}{30}$

$30 \cdot \sin 45 = y$

$x = 21.21 = y$

Dotted

$x = 15$

$y = 26$

c. Identify three other locations on the circle that would have the same y-values as the ones you just found. What central angles would you use to show those locations?

They are in quadrant II. You just flip the triangles over the y-axis. Angles are 150°, 135°, & 120°

d. Identify three other locations on the circle that would have the same x-values as the ones you found in part (b). What central angles would you use to show those locations?

Quadrant IV. Flip triangles over x-axis. Angles are 300°, 315°, 330° (OR -60°, -45°, -30°)

8. How can you convert the xy coordinates from problem 7 into xy coordinates for Bertha's circle on the front page?

Keep x the same, subtract 50 for y.