Name:_____ Date:_____

Using Trig to Understand Gears

Gears are interlocking circles that rotate together with teeth around the circumference. As one gear turns, it automatically turns all the other gears it touches. Keep in mind that the direction of rotation changes (if one gear turns clockwise, it will turn the next gear counterclockwise).

1. In the video, the two gears have a ratio of 40 teeth: 20 teeth, or 2:1. The bigger gear, (the blue one on the right) moves clockwise, and the smaller gear rotates counterclockwise.

a.

First, fill out this table. Keep in mind that the two gears rotate in opposite directions. Use degrees.

Small Gear Angle	Big Gear Angle	Sin(Small Gear Angle)	Sin(Big Gear Angle)
0	0	0	0
90	-45	1	$-\frac{\sqrt{2}}{2}$
180	-90	0	-1
270	-135	-1	$-\frac{\sqrt{2}}{2}$
360	-180	0	0

If θ rotates counterclockwise, then $-\theta$ rotates clockwise

b. Now, let x = Small Gear Angle. Write an equation for f(x) and g(x), where f(x) is the sine of the small gear and g(x) is the sine of the big gear. Verify your functions using a calculator.

$f(x) = \sin(x)$

$g(x) = \sin(-\frac{x}{2})$

The negative makes it rotate the other way. The $\div 2$ makes it half as fast.

c. Where does the original gear ratio, 2:1, show up in your functions f and g?

2:1 is like $\frac{1}{2}$, which is inside of g(x)

d. But the two gears also have different radii, so it does not make sense for their functions to have the same amplitude. Use the gear ratio to fix your functions f and g so they model the gears better. Assume the small gear has a radius of 1 unit.

$f(x) = 1 \cdot \sin(x)$

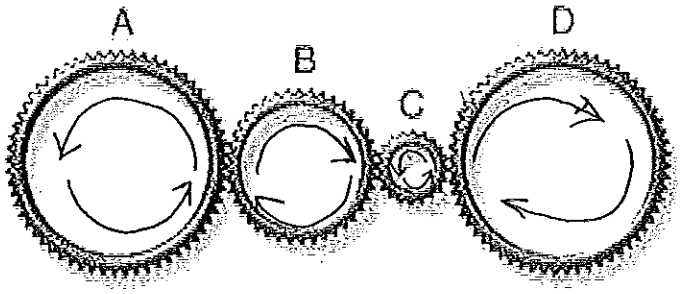
$g(x) = 2 \cdot \sin(-\frac{x}{2})$

Remember that a = amplitude = radius

2. Sometimes gears get linked together in a chain, like the four gears in the video. Use the picture below as a guide.

The gear ratio is as follows:

A:B:C:D
6:3:1:6



a. Imagine turning gear A counterclockwise. Then gear A is called the driver, and the other gears "follow" its rotation. What direction do gears B, C, and D rotate, respectively?

Clockwise, counterclockwise, clockwise

b. Which gear will spin the fastest? How much faster does it spin than the slowest gear?

Gear C because it is the smallest. It spins 6 times faster.

c. Let x = degrees of rotation of gear A. Write functions that describe the rotation of each gear in terms of the rotation of gear A. Make sure you have the correct orientation, amplitude, and spin rate. Assume the smallest gear has a radius of 1.

$A(x) = 6 \cdot \sin(x)$ $\quad D(x) = 6\sin(x)$

$B(x) = 3 \cdot \sin(-2x)$

$C(x) = \sin(6x)$

d. If x = degrees of rotation of gear A, what is the period of gear A? What is the period of gear B? Gear C? Gear D?

A: 360° $\quad$ D: 360°

B: 180°

C: 60°

e. How do the periods compare to the functions for each gear? Can you tell the period just by looking at the function?

Each period is just $\frac{360}{b}$ where b is the number in front of x.