

Sketching Polynomials

Remember that polynomials come in two different forms:

Standard form

$$x^4 - 7x^3 + 7x^2 + 15x$$



These are called "terms"

because they are added

Factored Form

$$x \cdot (x-3)(x-5)(x+1)$$



These are called "factors"

because they are multiplied

The constants in front of each term is called a "coefficient."

The biggest power in the polynomial is called the "degree"

If a polynomial is in factored form, the zeros/roots are the opposites of each factor.

$$x(x-3)(x-5)(x+1)$$

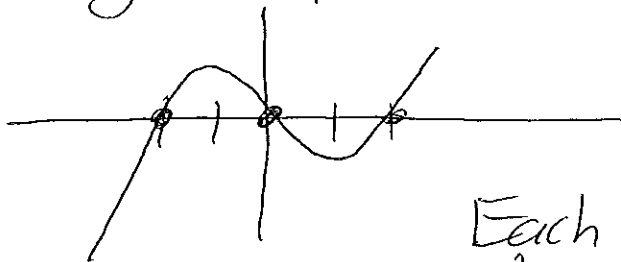
Roots: $x=0, x=3, x=5, x=-1$

Furthermore, the degree is 4, which is even. All even degree polynomials look roughly parabolic (with some extra wiggles).

Remember that the power of each factor determines the behavior of the polynomial near that root. Look at these 3 examples.

$$f(x) = x \cdot (x+2)(x-2)$$

degree = 3, looks cubic

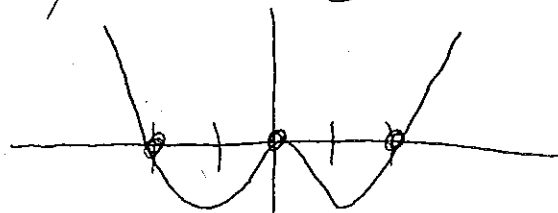
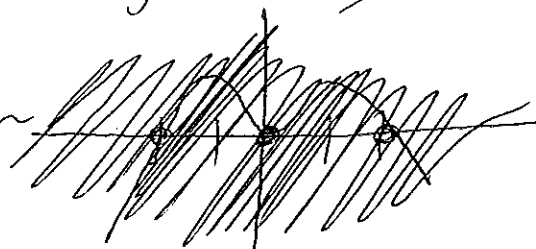


Each factor was linear, each root was crossed by a line.

$$g(x) = x^2 \cdot (x+2)(x-2)$$

degree = 4, looks parabolic

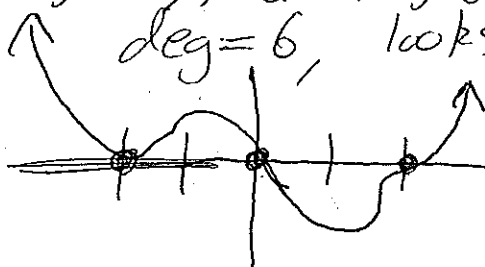
Wrong orientation



The squared factor makes its root look parabolic

$$h(x) = x \cdot (x+2)^2(x-2)^3$$

deg = 6, looks parabolic



Each root looks like its factor.