Imaginary Numbers

Note: This page only covers operations with imaginary & complex numbers.

See "Number Systems" for the theoretical & historical background.

See "Complex Roots" for the connection to polynomials, and how to visualize.

"i" is an imaginary number.

It is defined as $i := \sqrt{-1}$

Thus, $i^2 = -1$

Theorem: The powers of i are cyclic.

$i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

x	0	1	2	3	4	5	6	7	8
i^x	1	i	-1	$-i$	1	i	-1	$-i$	1

See how it cycles with a period of 4?

Adding/Subtracting complex numbers:
Just combine the real & imaginary parts, similar to adding like terms.

1) $(3+2i)+(5-4i)$

$(3+5)+(2i-4i) = 8-2i$

2) $(7+3i)-(5-2i)$

$(7-5)+(3i-(-2i))$

$2+5i$

Multiplying: Use the distributive property (FOIL), substitute $i^2=-1$, then combine real & imaginary parts.

1) $(-2+3i)\cdot(4+2i)$

$-8-4i+12i+6i^2$

$-8+8i+6(-1)$

$-14+8i$

2) $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$\frac{1}{4}+\cancel{\frac{\sqrt{3}}{4}i}-\cancel{\frac{\sqrt{3}}{4}i}-\frac{3}{4}i^2$

$\frac{1}{4}-\frac{3}{4}(-1)=1.$ or $1+0i$

Conjugate: The conjugate has the same real part, but the opposite imaginary part. Write $\bar{z}$, say "zee bar".

1) If $z=3+4i$, $\bar{z}=3-4i$

2) If $z=-2-4i$, $\bar{z}=-2+4i$

3) If $z=1$, $\bar{z}=1$

4) If $z=i$, $\bar{z}=-i$