

# Polynomial Division

Remember that multiplication & division are inverses. If you understand multiplication, you can work backwards to divide.

Multiplying: Distributive Property

$$(x^3 + 2x^2 + 3x - 7)(x^2 + x + 1)$$

$$x^5 + x^4 + x^3 + 2x^4 + 2x^3 + 2x^2 + 3x^3 + 3x^2 + 3x - 7x^2 - 7x - 7$$

$$x^5 + 3x^4 + 6x^3 - 2x^2 - 4x - 7$$

Generic Rectangle

|       |                                     |                       |        |         |  |
|-------|-------------------------------------|-----------------------|--------|---------|--|
|       |                                     | $x^3 + 2x^2 + 3x - 7$ |        |         |  |
| $x^2$ | $x^5$                               | $2x^4$                | $3x^3$ | $-7x^2$ |  |
| $+x$  | $x^4$                               | $2x^3$                | $3x^2$ | $-7x$   |  |
| $+1$  | $x^3$                               | $2x^2$                | $3x$   | $-7$    |  |
|       | $x^5 + 3x^4 + 6x^3 - 2x^2 - 4x - 7$ |                       |        |         |  |

← The like terms are on each diagonal

Use whichever method you like. The generic rectangle is easier to use for division, in my opinion.

Dividing: Generic Rectangle

|      |                        |                        |       |      |
|------|------------------------|------------------------|-------|------|
|      |                        | $x^3 + 2x^2 - 5x + 12$ |       |      |
|      |                        | $x + 4$                |       |      |
|      |                        | $x^2$                  | $-2x$ | $+3$ |
| $x$  | $x^3$                  | $-2x^2$                | $3x$  |      |
| $+4$ | $4x^2$                 | $-8x$                  | $12$  |      |
|      | $x^3 + 2x^2 - 5x + 12$ |                        |       |      |

So,  $x^2 - 2x + 3 = \frac{x^3 + 2x^2 - 5x + 12}{x + 4}$

# Polynomial Division

Long Division: Match biggest terms, multiply, then subtract.

$$\begin{array}{r}
 x^2 - 2x + 3 \\
 x + 4 \overline{) x^3 + 2x^2 - 5x + 12} \\
 \underline{-(x^3 + 4x^2)} \phantom{+ 12} \\
 -2x^2 - 5x \phantom{+ 12} \\
 \underline{-(-2x^2 - 8x)} \phantom{+ 12} \\
 3x + 12 \\
 \underline{-(3x + 12)} \\
 0
 \end{array}$$

Sometimes polynomials don't divide evenly. Just use a remainder.

$$\begin{array}{r}
 x^2 + 7x + 38 \\
 x - 5 \overline{) x^3 + 2x^2 + 3x + 4} \\
 \underline{-(x^3 - 5x^2)} \phantom{+ 4} \\
 7x^2 + 3x \phantom{+ 4} \\
 \underline{-(7x^2 - 35x)} \phantom{+ 4} \\
 38x + 4 \\
 \underline{-(38x + 190)} \\
 -186
 \end{array}$$

$$\text{So, } \frac{x^3 + 2x^2 + 3x + 4}{x - 5} = x^2 + 7x + 38 + \frac{-186}{x - 5}$$

$$\text{OR} \\
 x^2 + 7x + 38 + \frac{-186}{x - 5}$$

|      |                       |        |        |
|------|-----------------------|--------|--------|
|      | $x^2 + 7x + 38$       |        |        |
| $x$  | $x^3$                 | $7x^2$ | $38x$  |
| $-5$ | $-5x^2$               | $-35x$ | $-190$ |
|      | $x^3 + 2x^2 + 3x + 4$ |        |        |

4 should go in here.

$$\text{But } 4 - 190 = -186.$$

$$\text{So } \frac{x^3 + 2x^2 + 3x + 4}{x - 5} = x^2 + 7x + 38 + \frac{-186}{x - 5}$$