Integral Roots Theorem.

If I start with a polynomial in factored form, then I use the distributive property to multiply it out.

$\boxed{Ex}$ 1) $(x-3)(x+5) = x^2+2x-15$

2) $(x-1)(x+1)(x-3) = (x^2-1)(x-3)$
x^3-3x^2-x+3

3) $(x-1)(x+1)(x-2)(x+2)(x-7)$
$(x^2-1)(x^2-4)(x-7)$
$(x^4-5x^2+4)(x-7)$
$x^5-7x^4-5x^3+35x^2+4x-28$

I gnore all variable terms. The constant term (AKA a_0) is always the product of the roots.

So if you start with a polynomial in standard form, the only possible integer roots are factors of a_0.

$\underline{Ex}$ 1) $f(x)=x^3-7x^2-x+7$

Possible roots: $\pm 1, 7 \longrightarrow$ Factors: $(x \pm 1)$ or $(x \pm 7)$

Plug in possible roots

$f(1)=0$ $\quad$ $f(7)=0$
$f(-1)=0$ $\quad$ $f(-7)\neq 0$

So $f(x)=(x-1)(x+1)(x-7)$

Sometimes you won't find all roots with IRT because there are irrational or complex roots. Use division.

Ex) $x^3-11x^2+x-11=g(x)$

Possible roots: $\pm 1, 11$.

$g(1)\neq 0$ $\quad g(-11)\neq 0$

$g(-1)\neq 0$ $\quad g(11)=0. \longrightarrow (x-11)$ factor

$$\begin{array}{r}
x^2 \quad + 1 \\
x-11\overline{)\,x^3-11x^2+x-11} \\
-(x^3-11x^2) \downarrow \quad \downarrow \\
\hline
0 \quad x-11 \\
-(x-11) \\
\hline
0
\end{array}$$

$= (x-11)(x^2+1)$

No real solution

So $x^3-11x^2+x-11=(x-11)(x+i)(x-i)$

Ex 2) $h(x)=x^4-10x^3+22x^2+30x-75$

Possible roots: $\pm 1, 3, 25, 75$ AND $5, 15$

$h(1)\neq 0$ $\quad h(3)\neq 0$ $\quad h(5)=0$ $\quad h(15)\neq 0$ etc.

$h(-1)\neq 0$ $\quad h(-3)\neq 0$ $\quad h(-5)\neq 0$ $\quad h(-15)\neq 0$

Only root: $x=5 \longrightarrow (x-5)$ factor.

	x^3	$-5x^2$	$-3x$	$+15$
x	x^4	$-5x^3$	$-3x^2$	$15x$
-5	$-5x^3$	$25x^2$	$15x$	-75

$x^4-10x^3+22x^2+30x-75$ ✓

$(x-5)(x^3-5x^2-3x+15)$

$\boxed{\text{Ex 2 cont'd}}$ So now you have a smaller degree polynomial. Use IRT on that new polynomial.

$$q(x) = x^3 - 5x^2 - 3x + 15$$

Possible roots: $\pm 1, 3, 5, 15$.

$q(1) \neq 0$ $\quad q(3) \neq 0$ $\quad q(5) = 0$ $\quad q(15) \neq 0$

$q(-1) \neq 0$ $\quad q(-3) \neq 0$ $\quad q(-5) \neq 0$ $\quad q(-15) \neq 0$.

$x = 5$ root (again), so $(x-5)$ factor

	x^2	0	-3
x	x^3	0	$-3x$
-5	$-5x^2$	0	15

$x^3 - 5x^2 - 3x + 15$ ✓

So the original polynomial can be factored:

$$h(x) = x^4 - 10x^3 + 22x^2 + 30x - 75$$

$$(x-5)(x-5)(x^2-3)$$

OR

$$(x-5)^2(x+\sqrt{3})(x-\sqrt{3})$$

That's it. Just keep dividing by factors until you have a quadratic factor. Then use the quadratic formula to solve.