Polynomial Review Packet

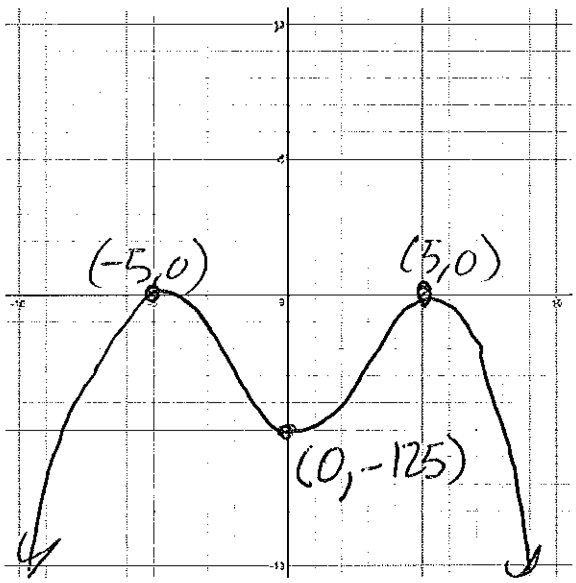
Name:______________________

1. Sketch a graph of each polynomial. Label at least one (x,y) point that is not a root.

$f(0) = -.2 \cdot 25 \cdot 25$

$= -125$

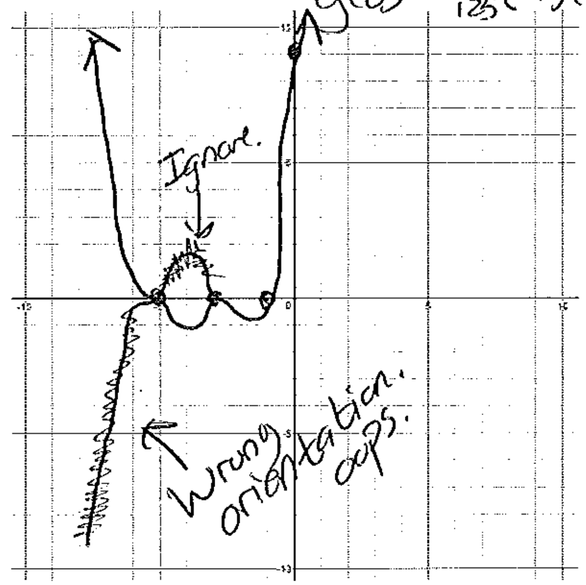
a. $f(x) = -0.2(x-5)^2(x+5)^2$



b. $g(x) = \frac{1}{125}(x+1)(x+3)^2(x+5)^3$

$g(0) = \frac{1}{125}(1)(9)(125)$

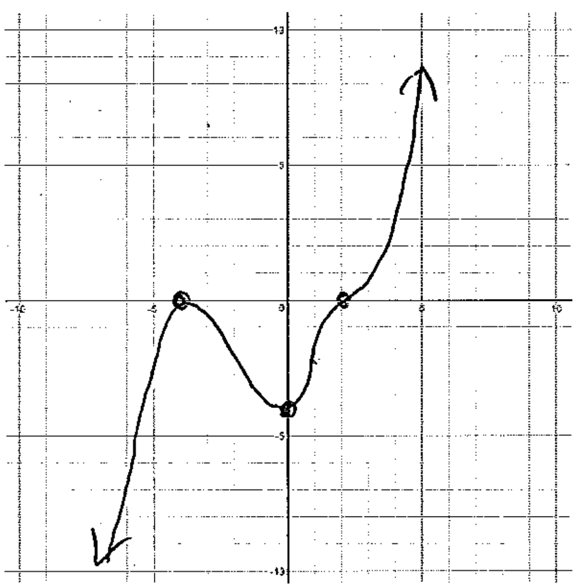
$= 9$



$h(0) = \frac{1}{32}(-8)(16)$

$= -4$

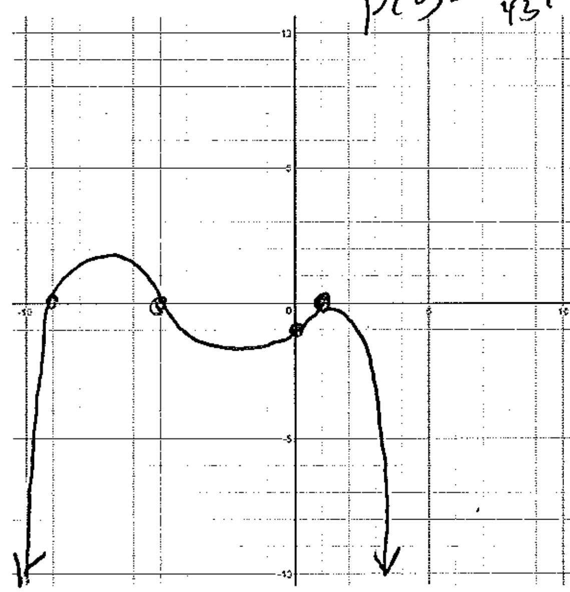
c. $h(x) = \frac{1}{32}(x-2)^3(x+4)^2$



d. $p(x) = \frac{-1}{45}(x+9)(x+5)(x-1)^2$

$p(0) = \frac{-1}{45}(9)(5)(1)$

$= -1$

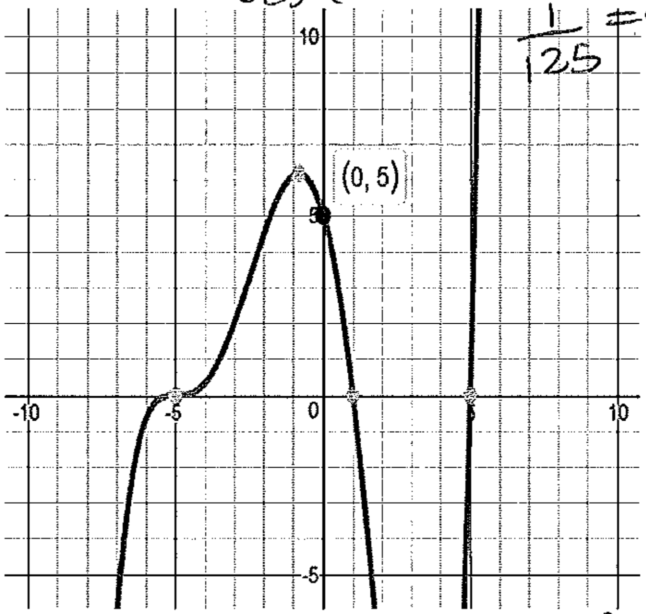


2. Find the exact equation of each polynomial, given a graph.

a. $f(x) = a(x+5)^3(x-1)(x-5)$

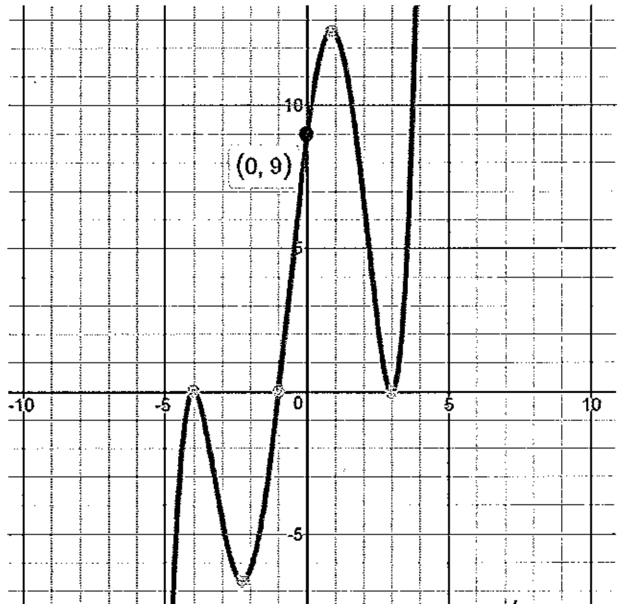
$5 = a(5)^3(-1)(-5) \rightarrow 5 = a \cdot 5^4$

$\frac{1}{125} = a$



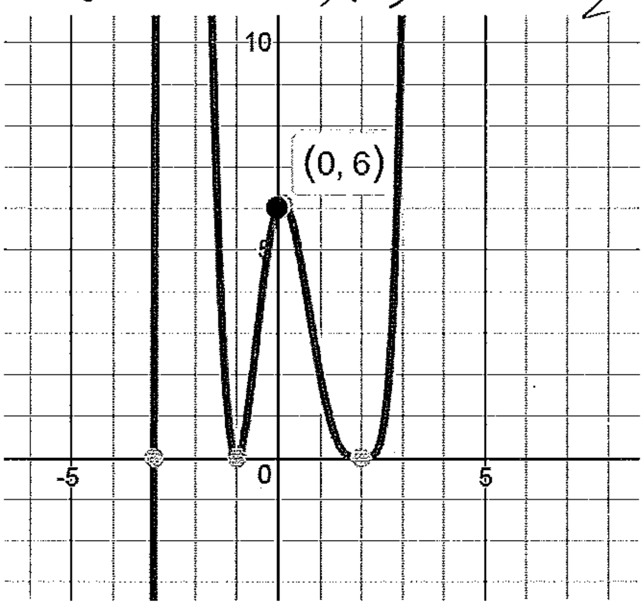
b. $g(x) = a(x+4)^2(x+1)(x-3)^2$

$9 = a(4)^2(1)(9) \rightarrow a = \frac{1}{16}$



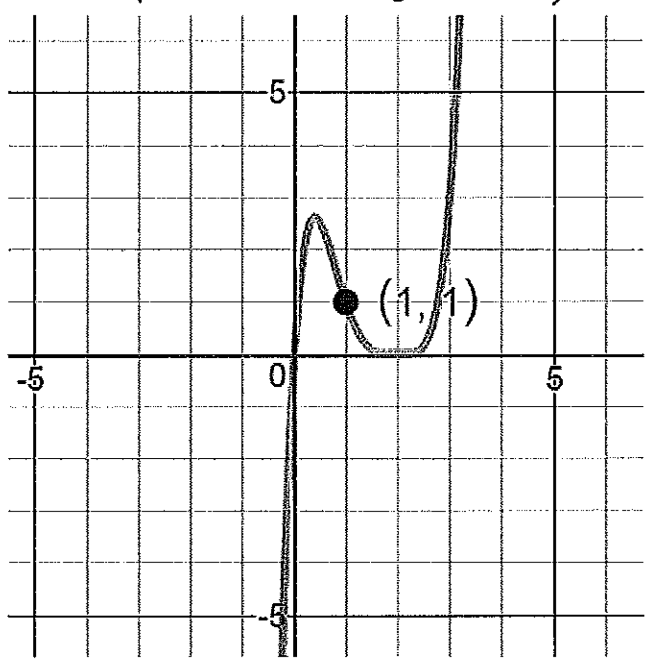
c. $h(x) = a(x+3)(x+1)^2(x-2)^2$

$6 = a(3)(1)(4) \rightarrow a = \frac{1}{2}$



d. $K(x) = a \cdot x \cdot (x-2)^4$

$1 = a \cdot 1 \cdot (-1)^4 \quad a = 1$



3. Find the exact equation of each polynomial, given a verbal description.

a. Cubic function, real root of 7, complex roots of 1+i, 1-i.

$$c(x) = (x-7)(x-(1+i))(x-(1-i))$$
$$(x-7)(x-1-i)(x-1+i)$$
$$(x-7)(x^2-2x+2) = x^3-2x^2+2x$$
$$-7x^2+14x-14$$
$$= x^3-9x^2+16x-14$$

b. Quartic function, complex roots of 3 + i, 3 - i, -2 + i, -2 - i

$$Q(x) = (x-(3+i))(x-(3-i))(x-(-2+i))(x-(-2-i))$$
$$(x-3-i)(x-3+i)(x+2-i)(x+2+i)$$
$$(x^2-6x+10)(x^2+4x+5) = x^4+4x^3+5x^2-6x^3-24x^2-30x+10x^2+40x+50$$
$$= x^4-2x^3-9x^2+10x+50$$

c. Quintic function (5th degree), real roots of 1, 2, 3, complex roots of 5+ i and its conjugate

$$Q(x) = (x-1)(x-2)(x-3)(x-(5+i))(x-(5-i))$$
$$(x-1)(x-2)(x-3)(x-5-i)(x-5+i)$$
$$(x-1)(x-2)(x-3)(x^2-10x+26)$$
$$(x^2-3x+2)(x-3)(x^2-10x+26)$$
$$(x^3-3x^2+2x-3x^2+9x-6)(x^2-10x+26)$$
$$(x^3-6x^2+11x-6)(x^2-10x+26)$$
$$x^5-10x^4+26x^3-6x^4+60x^3-156x^2+11x^3-110x^2+286x-6x^2+60x-156$$
$$= x^5-16x^4+97x^3-272x^2+346x-156$$

4. Completely factor the polynomial p(x) = $x^4 - 2x^3 - 4x^2 - 8x - 32$, given that two of the roots are 4 and -2

$$\begin{array}{r} x^3+2x^2+4x+8 \\ x-4\,\overline{)\,x^4-2x^3-4x^2-8x-32} \\ -(x^4-4x^3) \\ 2x^3-4x^2 \\ -(2x^3-8x^2) \\ 4x^2-8x \\ -(4x^2-16x) \\ 8x-32 \\ -(8x-32) \\ 0 \end{array}$$

$$\begin{array}{r} x^2+4 \\ x+2\,\overline{)\,x^3+2x^2+4x+8} \\ -(x^3+2x^2) \\ 0 \quad 4x+8 \\ -(4x+8) \\ 0 \end{array}$$

So, $p(x) = (x-4)(x+2)(x^2+4)$

OR

$p(x) = (x-4)(x+2)(x+2i)(x-2i)$

5. Divide the polynomials

a. $\dfrac{x^4+4x^3-5x^2-36x-36}{x+7}$

	x^3	$-3x^2$	$16x$	-148	R
x	x^4	$-3x^3$	$16x^2$	$-148x$	1000
7	$7x^3$	$-21x^2$	$112x$	-1036	

$$x^4+4x^3-5x^2-36x-36$$

$$= x^3-3x^2+16x-148+\frac{1000}{x+7}$$

b. $\dfrac{x^3-9x^2+24x-20}{x+9}$

	x^2	$-18x$	186	R
x	x^3	$-18x^2$	$+186x$	-1694
9	$9x^2$	$-162x$	1674	

$$x^3-9x^2+24x-20$$

$$= x^2-18x+186+\frac{-1694}{x+9}$$

6. Mr. Maurer claims that one of the 3rd roots of 8 is z = $-1+\sqrt{3}i$

a. Find |z|. $|z| = \sqrt{a^2+b^2} = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2.$

Notice that $2^3 = 8$.

b. Show by direct computation that $z^3 = 8$

$(-1+\sqrt{3}i)^3 = (-1+\sqrt{3}i)(-1+\sqrt{3}i)(-1+\sqrt{3}i)$

$(1-2\sqrt{3}i+3i^2)(-1+\sqrt{3}i)$

$(1-2\sqrt{3}i-3)(-1+\sqrt{3}i)$

$(-2-2\sqrt{3}i)(-1+\sqrt{3}i)$

$2-\cancel{2\sqrt{3}i}+\cancel{2\sqrt{3}i}-2\cdot3i^2$

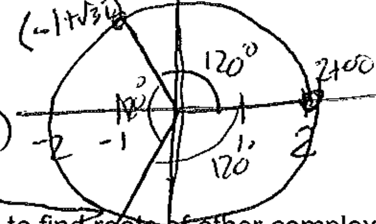
$2-6i^2$

$2+6=8.$

c. Make a geometric argument that $z^3 = 8$, and list the other two 3rd roots of 8. (Hint: We did roots of unity in class, which are on a circle of radius 1. What radius makes sense for $z^3=8$?)

$|z| = 2 = r$

conjugate! $\rightarrow$ $(-1-\sqrt{3}i)$



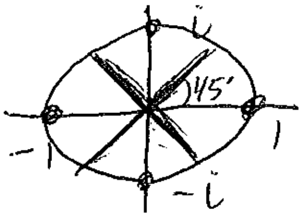
$z = -1+\sqrt{3}i$

$z = -1-\sqrt{3}i$

$z = 2+0i$

7. You can also use a similar geometric argument to find roots of other complex numbers.

a. Find the four 4th roots of -1. -1 has angle $180°$.



$180°/4 = 45°.$

$\cos(45) = \frac{\sqrt{2}}{2} = \sin(45)$

So $z = \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i.$

Other roots are evenly spaced around circle. So $90°$ apart.

Thus, $z = -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$

$z = -\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

$z = \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

b. Show by direct computation that $z^4 = -1$ for one of your complex roots.

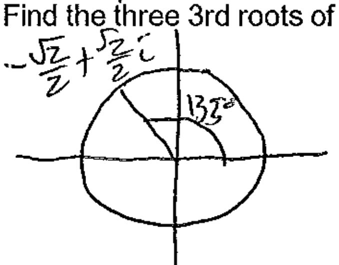
Let $z = \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$

$z^4 = (\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)$

$(\frac{2}{4}+2\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}i+\frac{2}{4}i^2)^2 = (\frac{1}{2}+\frac{2}{2}i-\frac{1}{2})^2 = (i)^2 = -1.$

:)

c. Find the three 3rd roots of $\frac{-\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$



$135°/3 = 45°.$

So $z = \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$

Other roots are evenly spaced, so $120°$ apart

$45+120 = 165$

$165+120 = 285$

d. Show by direct computation that $z^3 = \frac{-\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$ for one of your complex roots.

$z_{165°} = -.966+.259i$

$z_{285°} = .259-.966i$

We know from (b) that if

$z = \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$, then $z^2 = i$.

So $z^3 = z^2 \cdot z = i\cdot(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i) = \frac{\sqrt{2}}{2}i+\frac{\sqrt{2}}{2}i^2 = -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$

:)

12. The roots of the polynomial g(x) are in a geometric sequence where $a_1 = 24$, $m = 0.5$, $n = 4$. Use the fact that $f(0) = 1$ to find the exact equation of f(x).

$g(x) = a(x-24)(x-12)(x-6)(x-3)$

$1 = a(-24)(-12)(-6)(-3)$

$g(x) = \frac{1}{5184}(x-24)(x-12)(x-6)(x-3)$

13. a. Completely factor $p(x) = x^5 - 12x^4 + 25x^3 + 50x^2 + 84x + 392$. Use the Integral Roots Theorem and polynomial division.

Possible roots: $392 = 49 \cdot 8 = 7^2 \cdot 2^3$

$p(-2) = 0$

	x^4	$-14x^3$	$+53x^2$	$-56x$	$+196$
x	x^5	$-14x^4$	$53x^3$	$-56x^2$	$196x$
$+2$	$2x^4$	$-28x^3$	$106x^2$	$-112x$	392

$x^5 - 12x^4 + 25x^3 + 50x^2 + 84x + 392$

	x^3	$-7x^2$	$+4x$	-28
x	x^4	$-7x^3$	$4x^2$	$-28x$
-7	$-7x^3$	$49x^2$	$-28x$	196

$x^4 - 14x^3 + 53x^2 - 56x + 196$

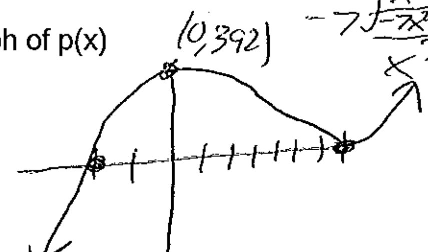
$p(7) = 0$

	x^2	0	4
x	x^3	0	$4x$
-7	$-7x^2$	0	-28

$x^3 - 7x^2 + 4x - 28$

So, $p(x) = (x-7)^2(x+2)(x^2+4)$

b. Sketch a graph of p(x)



14. a. Completely factor $q(x) = 2x^4 + 2x^3 + 14x^2 + 18x - 36$. Use the Integral Roots Theorem and polynomial division. Possible Roots: $\pm 1, 2, 3, 4, 6, 9, 12, 18, 36$

$q(1) = 0$

	$2x^3$	$+4x^2$	$+18x$	$+36$
x	$2x^4$	$4x^3$	$18x^2$	$36x$
-1	$-2x^3$	$-4x^2$	$-18x$	-36

$2x^4 + 2x^3 + 14x^2 + 18x - 36$

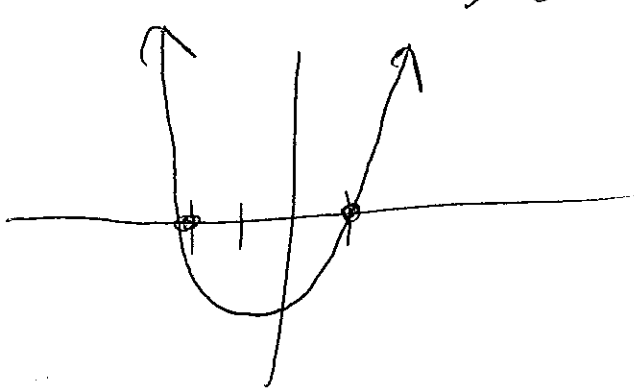
$q(-2) = 0$

	$2x^2$	0	18
x	$2x^3$	0	$18x$
$+2$	$4x^2$	0	36

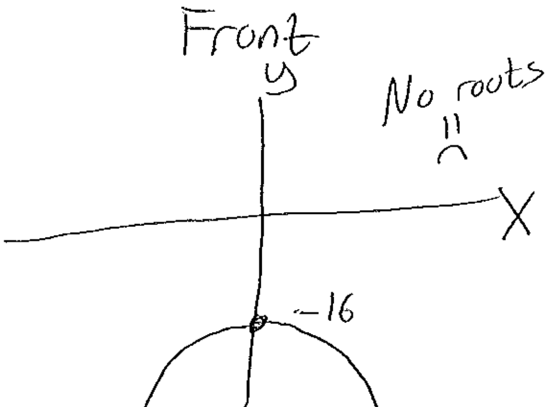
$2x^3 + 4x^2 + 18x + 36$

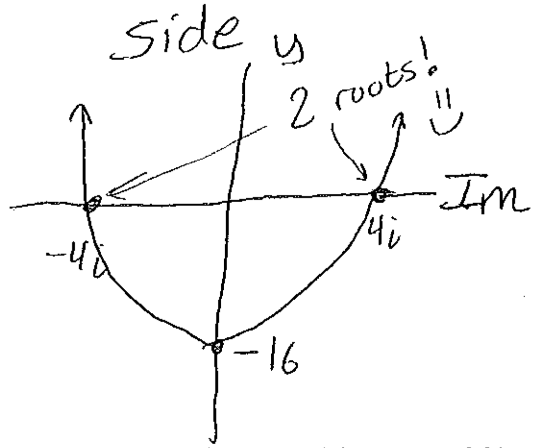
So, $q(x) = (x-1)(x+2)(2x^2+18)$

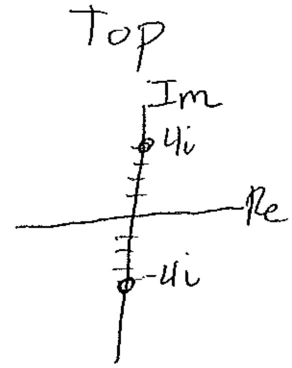
b. Sketch a graph of q(x)



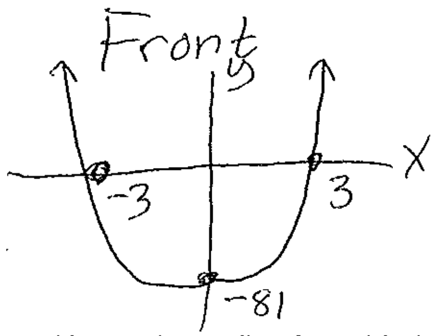
8. The function $f(x) = -1x^2 - 16$ does not seem to have any roots. Sketch 3 different view of f(x) to show where the roots are. Make sure to label your axes and any important points.

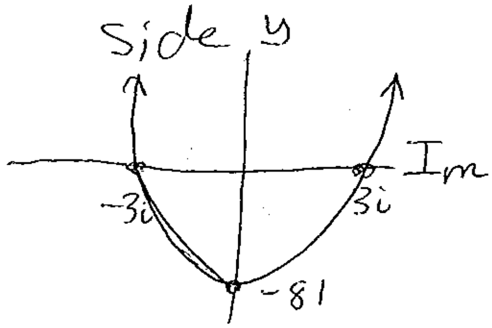


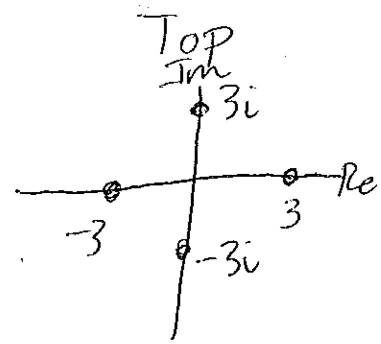




9.The function $p(x) = x^4 - 81$ has two real roots. The Fundamental Theorem of Algebra states that any polynomial of degree *n* must have exactly *n* roots. Sketch 3 different views of p(x) to show where all *n* roots are. Label your axes and any important points.







10. a. Is it possible for a 4th degree polynomial to have 1 real root? Explain.

Yes. x^4 has only $x=0$

b. Is it possible for a 4th degree polynomial to have 2 real roots? Explain.

Yes. $x^3 \cdot (x-1)$ has only $x=0$ & $x=1$.

c. Is it possible for a 4th degree polynomial to have 0 real roots? Explain.

Yes. $x^4 + 99999$ has no real roots.

11. The roots of the polynomial f(x) are in an arithmetic sequence where $a_1 = -2$, $d = 4$, $n = 3$. Use the fact that $f(0) = 12$ to find the exact equation of f(x).

$f(x) = a(x+2)(x-2)(x-6)$

$12 = a(2)(-2)(-6)$

$12 = a \cdot 24$

$\frac{1}{2} = a$

$f(x) = \frac{1}{2}(x+2)(x-2)(x-6)$