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Algebra 3-4
SPRING FINAL 2016

Name: _____ Per:_____

1. Evaluate the following expressions.

a. $\log_4 16 + \log_2 32$ $(4^2=16,\ 2^5=32)$

$2+5=7$

b. $\log_{1/4} 16 - \log_{1/2} 32$ $\left(\left(\frac{1}{4}\right)^{-2}=16,\ \left(\frac{1}{2}\right)^{-5}=32\right)$

$-2--5=3$

b. $\log_7 49 - \log_{1/2}\left(\frac{1}{4}\right)$ $\left(7^2=49, \left(\frac{1}{2}\right)^2=\frac{1}{4}\right)$

$2-2=0$

b. $\log_5 25 - \log_{12}(1)$ $5^2=25,\ 12^0=1$

$2-0=2$

2. Use your Unit Circle to find 2 angles in radians for the domain $0 \le \theta < 2\pi$.

a. $\sin\theta = \frac{-\sqrt{3}}{2}$ $\frac{4\pi}{3}, \frac{5\pi}{3}$

b. $\cos\theta = \frac{1}{2}$ $\frac{\pi}{3}, \frac{5\pi}{3}$

c. $\sin\theta = \frac{-1}{2}$ $\frac{7\pi}{6}, \frac{11\pi}{6}$

d. $\cos\theta = \frac{-1}{2}$ $\frac{2\pi}{3}, \frac{4\pi}{3}$

3. Sketch graphs of each equation.

a. $y = 5\sin\left(x + \frac{\pi}{2}\right)$

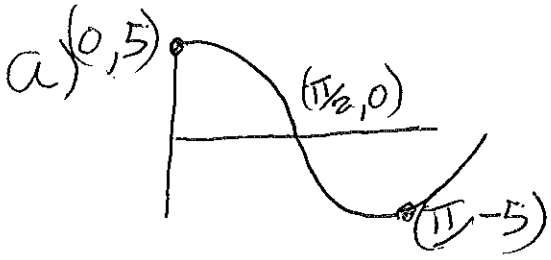
b. $y = 5\sin(x) - 2$

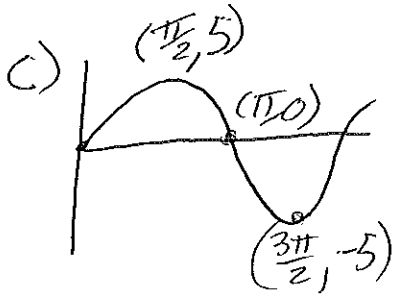
c. $y = 5\sin(x)$

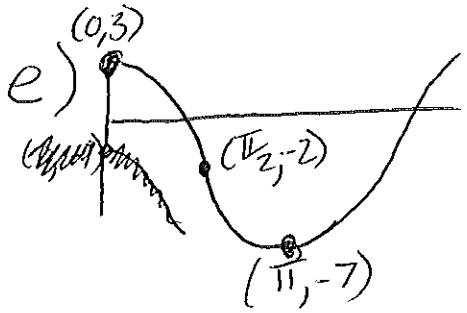
d. $y = \cos(x) + 5$

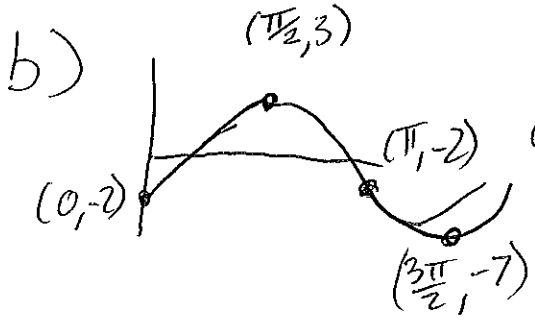
e. $y = 5\cos(x) - 2$

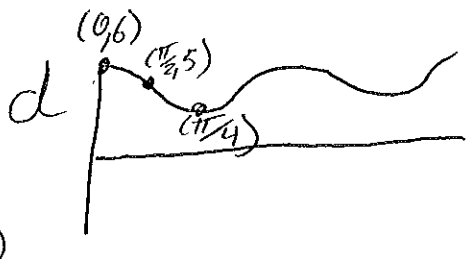
f. $y = 5\cos\left(x + \frac{\pi}{2}\right) + 2$

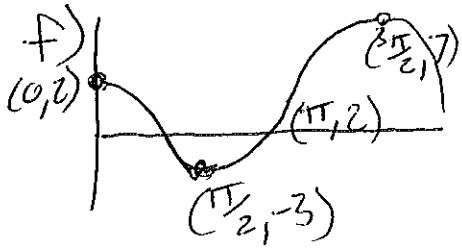












4. A news program reports on a proposed bill in the Oregon Legislature. The purpose of the program is to find out what percent of the population in its viewing area favors the proposed bill. Consider each of the following survey methods. For each one, explain any bias you can find. If you think it is unbiased (fair), explain why.

a. Viewers are invited to call into the program and express their preferences.

Voluntary Response Bias – Only those who care enough to call

b. A reporter interviews people on the street near the capitol building in Salem.

Sampling Bias – Leaves out lots of people

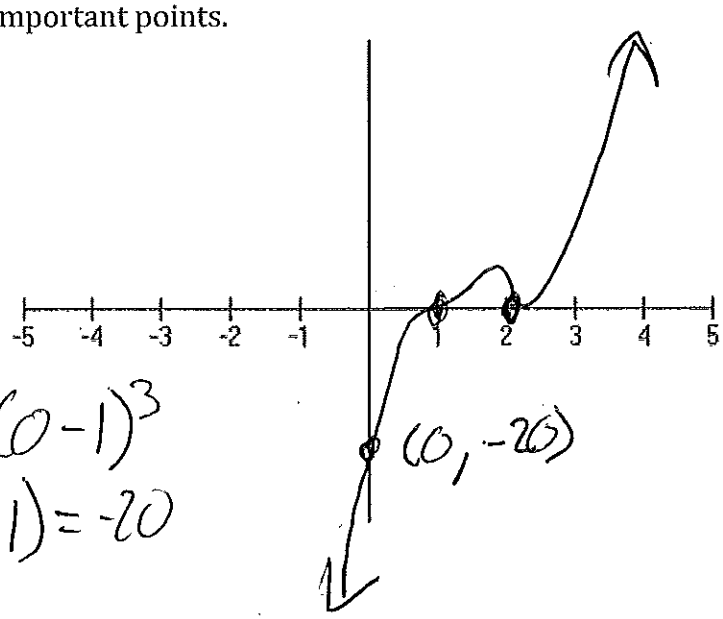
c. The news program posts a survey on Facebook asking "Do you support this bill?"

Sampling Bias & Voluntary Response bias.

d. The news program does a cluster sample of its viewing population and asks a representative sample the following: "The bill would increase funding for K-12 education by 10%. Do you support this bill?

Preface Bias.

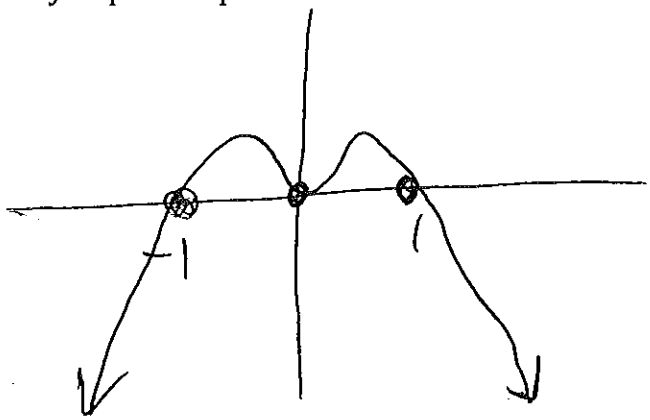
5. Sketch the graph of $y = 5(x-2)^2(x-1)^3$. Label any important points.



$5(0-2)^2(0-1)^3$

$5(4)(-1) = -20$

6. Sketch the graph of $y = -3(x)^2(x+1)(x-1)$. Label any important points.



7. Find the factored form of the polynomial using any method. Then find all of the roots. Sketch a graph.

$y = x^3 + 2x^2 - 4x - 8$

Possible Roots: $\pm 1, 2, 4, 8$

$2^3 + 2(2)^2 - 4(2) - 8$

$8 + 2 \cdot 4 - 8 - 8 = 0 \rightarrow$ 2 is a root

factored form: $(x-2)(x+2)^2$

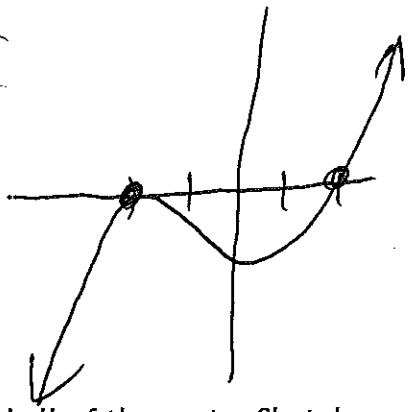
roots: $2, -2$

	x^2	$4x$	4
x	x^3	$4x^2$	$4x$
-2	$-2x^2$	$-8x$	-8

$x^3 + 2x^2 - 4x - 8$

$(x-2)(x^2+4x+4)$

$(x-2)(x+2)^2$



8. Find the factored form of the polynomial using any method. Then find all of the roots. Sketch a graph.

$y = x^4 - 10x^2 + 9$

Possible roots: $\pm 1, 3, 9$

$1^4 - 10(1)^2 + 9$

$1 - 10 + 9 = 0 \rightarrow$ 1 is a root

factored form: $(x-1)(x+1)(x-3)(x+3)$

roots: $\pm 1, \pm 3$

	x^3	x^2	$-9x$	-9
x	x^4	x^3	$-9x^2$	$-9x$
-1	$-x^3$	$-x^2$	$9x$	9

$(x-1)(x^3+x^2-9x-9)$

$3^3 + 3^2 - 9(3) - 9$

$27 + 9 - 27 - 9 = 0$

3 is a root.

	x^2	$4x$	3
x	x^3	$4x^2$	$3x$
-3	$-3x^2$	$-12x$	-9

$(x-1)(x-3)(x^2+4x+3)$

$(x-1)(x-3)(x+1)(x+3)$

Oops, no bounces

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Algebra 3-4 Name: _____ Per: ____

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DO NOT TRY TO CANCEL.

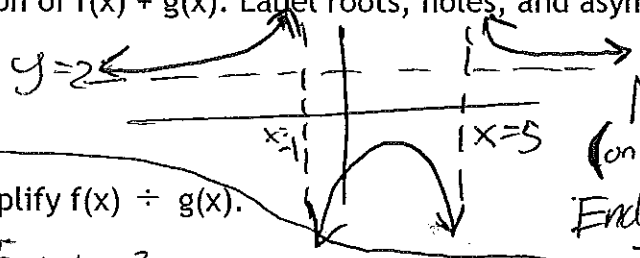
9. Let $f(x) = \frac{x-5}{x+1}$ and $g(x) = \frac{x-3}{x-5}$

a. Find and simplify f(x) + g(x).

$$\frac{(x-5)(x-5)}{(x-5)(x+1)} + \frac{(x-3)(x+1)}{(x-5)(x+1)} = \frac{(x-5)(x-5)+(x-3)(x+1)}{(x-5)(x+1)}$$

b. Sketch a graph of f(x) + g(x). Label roots, holes, and asymptotes.

$$\frac{x^2-10x+25+x^2-2x-3}{x^2-4x-5}$$



No roots (on calc)

$$\frac{2x^2-12x+22}{x^2-4x-5}$$

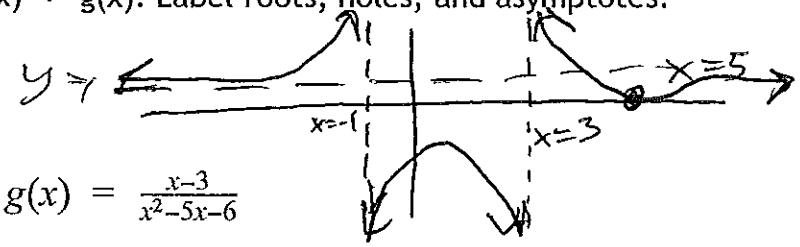
End Behavior = 2

c. Find and simplify f(x) ÷ g(x).

$$\frac{x-5}{x+1} \div \frac{x-3}{x-5} = \frac{x-5}{x+1} \cdot \frac{x-5}{x-3} = \frac{x^2-10x+25}{x^2-2x-3}$$

End Behavior = 1

d. Sketch a graph of f(x) ÷ g(x). Label roots, holes, and asymptotes.



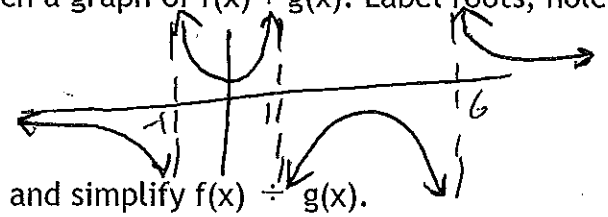
10. Let $f(x) = \frac{x-5}{x^2-1}$ and $g(x) = \frac{x-3}{x^2-5x-6}$

NOTHING CANCELS.

e. Find and simplify f(x) + g(x).

$$\frac{x-6}{x-6}\frac{x-5}{(x+1)(x-1)} + \frac{x-3}{(x-6)(x+1)}\frac{(x-1)}{(x-1)} = \frac{(x-6)(x-5)+(x-3)(x-1)}{(x-6)(x+1)(x-1)} = \frac{x^2-11x+30+x^2-4x+3}{(x^2-5x-6)(x-1)}$$

f. Sketch a graph of f(x) + g(x). Label roots, holes, and asymptotes.



No roots (on calc)

$$\frac{2x^2-15x+33}{x^3-6x^2-x+6}$$

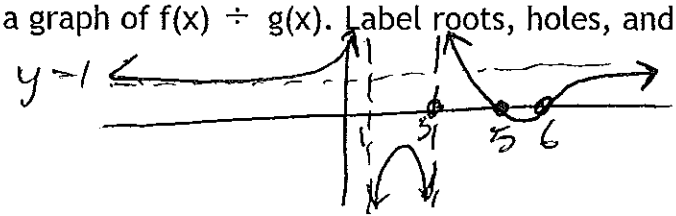
End Behavior = 0

g. Find and simplify f(x) ÷ g(x).

$$\frac{x-5}{(x+1)(x-1)} \div \frac{x-3}{(x-6)(x+1)} = \frac{x-5}{(x+1)(x-1)} \cdot \frac{(x-6)(x+1)}{x-3} = \frac{(x-5)(x-6)}{(x-1)(x-3)}$$

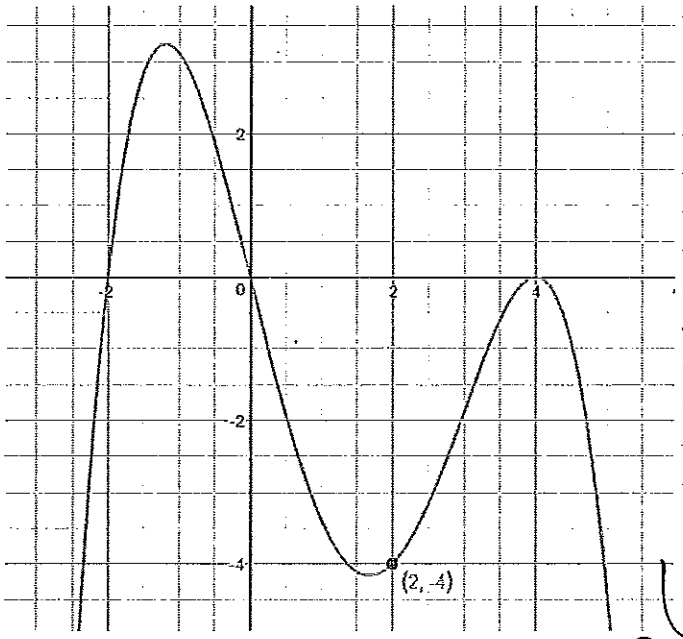
End Behavior = 1

h. Sketch a graph of f(x) ÷ g(x). Label roots, holes, and asymptotes.



11. Give the **exact** equation of the polynomial. Show all work!

a. $-\frac{1}{8}(x+2)x(x-4)^2$



$y = a(x+2)\cdot x(x-4)^2$

$-4 = a(2+2)2(2-4)^2$

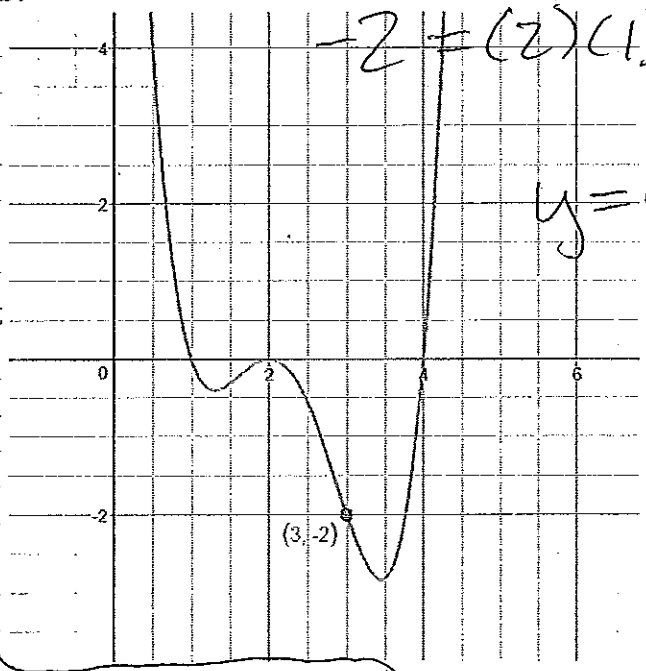
b. $y = (x-1)(x-2)^2(x-4)\cdot a$

$-2 = (3-1)(3-2)^2(3-4)a$

$-2 = (2)(1)(-1)a$

$a = 1$

$y = (x-1)(x-2)^2(x-4)$



$-4 = a(4)(2)(4)$

$-\frac{1}{8} = a$

12. a. Find all roots. $y = 2x^3 + 7x^2 - 18x - 63$

3 is root (from calc)

$(x-3)(2x^2+13x+21)$

	$2x^2$	$13x$	21
x	$2x^3$	$13x^2$	$21x$
-3	$-6x^2$	$-39x$	-63

$x = \frac{-13 \pm \sqrt{13^2 - 4(2)(21)}}{2(2)}$

$= \frac{-13 \pm \sqrt{169-168}}{4} = \frac{-13 \pm 1}{4}$

$x = \frac{-12}{4}, \frac{-14}{4}$

$-3, -3.5$

$x = 3$

$x = -3$

$x = -3.5$

b. Find all roots. $y = x^4 + 4x^3 - 13x^2 - 64x - 48$

-1 & 4 are roots (from calc)

Divide by $(x+1)(x-4) = x^2 - 3x - 4$

$x^2-3x-4 \overline{)\, x^4+4x^3-13x^2-64x-48}$

$-(x^4-3x^3-4x^2)$

$7x^3-9x^2-64x$

$-(7x^3-21x^2-28x)$

$12x^2+-36x-48$

$12x^2-36x-48$

$(x^2+7x+12)(x^2-3x-4)$

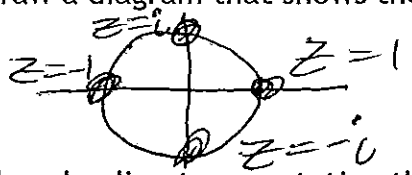
$x = -3$

$x = 4$

$x = -1, x = 4$

13. Mr. Maurer claims that $z = i$ is one of the 4th roots of unity.

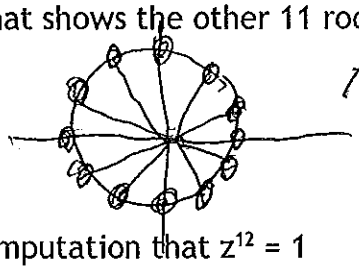
a. Draw a diagram that shows the other three roots of unity.



b. Show by direct computation that $z^4 = 1$ $\quad i^4 = i \cdot i \cdot i \cdot i = -1 \cdot -1 = 1$.

14. Mr. Maurer claims that $z = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ is one of the 12th roots of unity.

a. Draw a diagram that shows the other 11 roots of unity



All are on multiples of 30°.

$\frac{\sqrt{3}}{2}+\frac{1}{2}i$ $-\frac{\sqrt{3}}{2}+\frac{1}{2}i$ $-\frac{\sqrt{3}}{2}-\frac{1}{2}i$ $\frac{\sqrt{3}}{2}-\frac{1}{2}i$

$\frac{1}{2}+\frac{\sqrt{3}}{2}i$ $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ $\frac{1}{2}-\frac{\sqrt{3}}{2}i$

i -1 $-i$ 1

b. Show by direct computation that $z^{12} = 1$

$(\frac{\sqrt{3}}{2}+\frac{1}{2}i)(\frac{\sqrt{3}}{2}+\frac{1}{2}i)$ etc...

$(\frac{3}{4}+\frac{\sqrt{3}}{4}i+\frac{\sqrt{3}}{4}i+\frac{1}{4}i^2)6$

$(\frac{3}{4}+2\frac{\sqrt{3}}{4}i-\frac{1}{4})^6$

$(\frac{1}{2}+\frac{\sqrt{3}}{2}i)^6=(\frac{1}{2}+\frac{\sqrt{3}}{2}i)(\frac{1}{2}+\frac{\sqrt{3}}{2}i)$ etc

$(\frac{1}{4}+\frac{\sqrt{3}}{4}i+\frac{\sqrt{3}}{4}i+\frac{3}{4}i^2)^3$

$(\frac{1}{4}+2\frac{\sqrt{3}}{4}i-\frac{3}{4})=(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)^3$

$(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)$

$(\frac{1}{4}-\frac{\sqrt{3}}{4}i-\frac{\sqrt{3}}{4}i+\frac{3}{4}i^2)(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)$

$(-\frac{1}{2}-\frac{\sqrt{3}}{2}i)(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)$

$(\frac{1}{4}-\frac{\sqrt{3}}{4}i+\frac{\sqrt{3}}{4}i-\frac{3}{4}i^2)$

$(\frac{1}{4}-\frac{3}{4}(-1))$

$(\frac{1}{4}+\frac{3}{4})$

$1.$

11

15. Find the equation that matches each table of values

a.

0	undefined
1	0
5	1
25	2
125	3

x's multiply by 5, y's count up by 1.

$y = \log_5 x$

b.

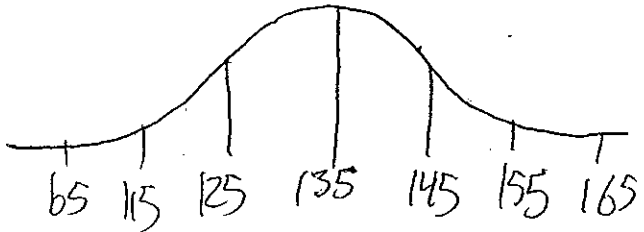
−1	−2.8
0	−2
1	2
2	22
3	122
4	622

$y = 5^x - 3$

These are almost powers of 5 $(1, 5, 25, 125, 625)$

15. A fifth grader takes a standardized achievement test that is normally distributed with a mean of 135 and a standard deviation of 10.

a. Sketch and label a normal curve.



b. If student A scores a 105 what is their z-score? $z = -3$

c. If student B scores a 145 what is their percentile rank?

$50+34=84^{th}$ percentile

d. What percent of students scored above a 155?

$\frac{100-95}{2} = \frac{5}{2} = 2.5\%$