

Day 20: Exploring Zero and Negative Exponents

Complete the table below.

Expression	Value (written as a whole number or fraction)
2^4	16
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2}$
2^{-2}	$\frac{1}{4}$
2^{-3}	$\frac{1}{8}$
2^{-4}	$\frac{1}{16}$

Zero Exponents:

$$x^0 = 1$$

Negative Exponents:

$$x^{-a} = \frac{1}{x^a}$$

Examples:

1. $x^{-2} \cdot x^5 = x^3$

2. $4^8 \cdot 4^{-3} = 4^5$

3. $3xy^3y^{-7} = 3xy^{-4} = \frac{3x}{y^4}$

4. $12ab^5b^{-11} = 12ab^{-6} = \frac{12a}{b^6}$

Practice: Negative and Zero Exponent Properties

1. Explain the rule of negative exponents in your own words:

Anything to a negative is a fraction.

For problems #2-6, simplify the expression (write it as just one base with one exponent):

2.) $3^3 \cdot 3^{-2}$ 3.) $6^7 \cdot 6^{-4}$ 4.) $7^{-3} \cdot 7^6$ 5.) $9^{-7} \cdot 9^9$ 6.) $11^3 \cdot 11^4 \cdot 11^{-4} \cdot 11^{-1}$

(Handwritten solutions: $3^1=3$, $6^3=216$, $7^3=343$, $9^2=81$, $11^2=121$)

Now for each of the above problems 2-6, go back and evaluate (find the value of) the expression. You may use your calculator if you need to.

2. 3. 4. 5. 6.

(Arrows point from the handwritten solutions above to these numbers)

For problems 7-11, evaluate each expression. Write your answer as a fraction. You should be able to do this without a calculator (although not necessarily in your head):

7.) 5^{-2} 8.) 7^{-3} 9.) 2^{-5} 10.) 3^{-4} 11.) 211^{-1}

(Handwritten solutions: $\frac{1}{5^2} = \frac{1}{25}$, $\frac{1}{7^3} = \frac{1}{343}$, $\frac{1}{2^5} = \frac{1}{32}$, $\frac{1}{3^4} = \frac{1}{81}$, $\frac{1}{211}$)

Simplify the following expressions. Your answers should NOT have any negative exponents once you are done simplifying.

12.) x^{-5} 13.) y^{-3} 14.) $h^5 \cdot h^{-4}$ 15.) $k^{-8} \cdot k^8$ 16.) $d^3 \cdot d^{-6} \cdot d^0$

(Handwritten solutions: $\frac{1}{x^5}$, $\frac{1}{y^3}$, $h^1 = h$, $k^0 = 1$, $d^{-3} = \frac{1}{d^3}$)

17.) $4^0 = ($ 18.) $x^0 = ($

~~MORE~~ Exponent Examples and Practice

Nasty

Combining Positive Properties

Example 1:

$$\begin{aligned} & (5x^3y^4)^2(2xy^3) \\ & 5^2(x^3)^2(y^4)^2 \cdot 2xy^3 \\ & 25x^6y^8 \cdot 2xy^3 \\ & 50x^7y^{11} \end{aligned}$$

Example 2:

$$\begin{aligned} & (-2xy^2)^2(4x^3y)^2 \\ & 4x^2y^4 \cdot 16x^6y^2 \\ & 64x^8y^6 \end{aligned}$$

Combining Negative Properties

Example 1:

$$\begin{aligned} & (3x^{-2}y)^2(3xy)^{-2} \\ & 9x^{-4}y^2 \cdot \frac{1}{9} \cdot \frac{1}{x^2} \cdot \frac{1}{y^2} \\ & = \frac{9 \cdot y^2}{9 \cdot x^4 \cdot x^2 \cdot y^2} = \frac{1}{x^6} \end{aligned}$$

Example 2:

$$\begin{aligned} & \frac{(4x^5y^{-4})^2}{2x^{-4}y^5} \\ & \frac{16x^{10}y^{-8}}{2x^{-4}y^5} \\ & 8x^{14}y^{-13} = \frac{8x^{14}}{y^{13}} \end{aligned}$$

You Try:

1. $(3x^4y^5)(2x^3)^2$

$$\begin{aligned} & 3x^4y^5 \cdot 4x^6 \\ & 12x^{10}y^5 \end{aligned}$$

2. $\frac{8a^5b^{-5}}{2a^{-2}b}$

$$4a^7b^{-6} = \frac{4a^7}{b^6}$$

Review

Use the **Product of Powers Property** (adding powers) to simplify each expression. You should have no numbers with exponents and no negative exponents.

1. $x^4 \cdot x^8$

$$x^{12}$$

2. $7x^9 \cdot x^3$

$$7x^{12}$$

3. $8x^3 \cdot 5x^3$

$$40x^6$$

4. $-2x^9(-6x^2)$

$$12x^{11}$$

5. $(3x^5y)(-4x^3y)$

$$-12x^8y^2$$

6. $9x^2y^3 \cdot 2x^2y$

$$18x^4y^4$$

7. $10x^2y^9 \cdot 3xy^6$

$$30x^3y^{15}$$

8. $5x^2y^7 \cdot 4x^3y$

$$20x^5y^8$$

Use the **Quotient of Powers Property** (subtracting powers) to simplify each expression. You should have no numbers with exponents and no negative exponents.

9. $\frac{x^{16}}{x^{12}}$

$$x^4$$

10. $\frac{x^7}{x^3}$

$$x^4$$

11. $\frac{8x^{10}}{2x^3}$

$$4x^7$$

12. $\frac{12x^6}{4x^5}$

$$3x^1$$

13. $\frac{6x^9y^8}{4x^3}$

$$1.5x^6y^8$$

14. $\frac{15x^2y^9}{10x^2y^4}$

$$1.5y^5$$

15. $\frac{30x^7y^4}{10x^4y^6}$

$$\frac{3x^3}{y^2}$$

16. $\frac{12x^4y^3}{8x^4y^5}$

$$= \frac{1.5}{y^2}$$