

Day 21: Fractional Exponent Investigation

I. Consider the following pattern:

A. Fill in the blanks based off of the examples.

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$$

$$2^3 = \underline{2} \cdot \underline{2} \cdot \underline{2} = \underline{8}$$

$$2^2 = \underline{2} \cdot \underline{2} = \underline{4}$$

$$2^1 = \underline{2}$$

$$2^0 = \underline{1}$$

$$2^{-1} = \underline{\frac{1}{2}}$$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{2} \cdot \frac{1}{2} = \underline{\frac{1}{4}}$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \underline{\frac{1}{8}}$$

$$2^{-4} = \frac{1}{2^4} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \underline{\frac{1}{16}}$$

B. Now consider $2^{\frac{1}{2}}$. Where would this fit in the pattern above? Draw an arrow where you think $2^{\frac{1}{2}}$ should be placed. What do you think the value will be?

My estimated value of $2^{\frac{1}{2}}$: 1.5

• Now enter $2^{\frac{1}{2}}$ in your calculator. What is the most specific number classification for the result?

• What is another key sequence on your calculator to find $2^{\frac{1}{2}}$? $\sqrt{2}$ Irrational

C. Now consider $2^{\frac{1}{3}}$. Where would this fit in the pattern above? Draw an arrow where you think $2^{\frac{1}{3}}$ should be placed. What do you think the value will be?

My estimated value of $2^{\frac{1}{3}}$: 1.3 Calculator value: 1.26 Number classification: Irrational

• What is another key sequence on your calculator to find $2^{\frac{1}{3}}$? $\sqrt[3]{2}$

D. Evaluate the following using your calculator:

$$36^{\frac{1}{2}} = 6$$

$$81^{\frac{1}{2}} = 9$$

$$64^{\frac{1}{2}} = 8$$

$$144^{\frac{1}{2}} = 12$$

$$25^{\frac{1}{2}} = 5$$

Write a statement about what the exponent $\frac{1}{2}$ represents.

The square root

Try to write this symbolically in radical form: $a^{\frac{1}{2}} = \sqrt{a}$

E. Based on your observations from part D, try to evaluate the following **without** your calculator.

$$8^{\frac{1}{3}} = 2$$

$$27^{\frac{1}{3}} = 3$$

$$1000^{\frac{1}{3}} = 10$$

$$125^{\frac{1}{3}} = 5$$

Write a statement about what the exponent $\frac{1}{3}$ represents?

Third (cube) root

Try to write this symbolically in radical form: $a^{\frac{1}{3}} = \sqrt[3]{a}$

F. Look back at parts D and E to complete the following symbolic rule in *radical form*:

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

II. Practice:

A. Rewrite in radical form, then simplify completely.

a. $100^{\frac{1}{2}} = \sqrt{100} = 10$

b. $125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

c. $17^{\frac{1}{2}} = \sqrt{17} \approx 4.12$

d. $64^{\frac{1}{3}} = 4$

e. $16^{\frac{1}{4}} = 2$

f. $16^{\frac{3}{4}} = 8$

g. $(8^{\frac{1}{2}})^2 = 8$

h. $(8^{\frac{1}{3}})^3 = 8$

i. $(8^4)^{\frac{1}{4}} = 8$

Notes: Fractional Exponents with a numerator $\neq 1$

Do the fraction, then raise to the numerator's power

B. Rewrite in exponential form, then simplify completely.

a. $\sqrt{81} = 9$

b. $\sqrt[3]{125} = 5$

c. $\sqrt[4]{20} =$

d. $\sqrt[3]{-64} =$

e. $\sqrt[3]{8} =$

f. $(\sqrt[3]{x})^4 =$

g. $(\sqrt{98})^2 =$

h. $(\sqrt[3]{98})^3 =$

i. $(\sqrt[4]{98})^4 =$