

In the Desmos activity, "Will it Hit the Hoop?", you investigated quadratic functions and their graphs. The graph of a quadratic equation describes the path of an object flying through the air.

1. Context

Deandre & Bree go down to the Cleveland football field to practice kicking field goals. Deandre films Bree kicking a football and they calculate that the equation for the football is $y = -16(x - 2)^2 + 64$, where x is the number of seconds since the ball has been kicked and y is the height of the football above the ground (in feet).

- a. Sketch a picture of the football's path through the air. Don't label it with any numbers, yet.



- b. What value represents the beginning of time? In other words, how do we represent the moment when Bree kicks the ball? $x=0$ is always the beginning

- c. What value represents the ground? In other words, how do we represent the height of the ground in the equation? $y=0$ is the ground.

- d. Mr. Maurer is curious about whether or not the football will go through the field goal or fall short. He knows the uprights are 10 feet high above the ground. Which equation below would help him answer the question? Explain how you know.

$$10 = -16(x - 2)^2 + 64$$

$$y = -16(10 - 2)^2 + 64$$

Because 10 feet high is a y -value, not a time.
(x)

- e. Ms. Muhs is curious about how high the ball was at the beginning of the kick (the moment when Bree kicked the ball). Which equation below would help her answer the question? Explain how you know.

$$0 = -16(x - 2)^2 + 64$$

$$y = -16(0 - 2)^2 + 64$$

Because $x=0$ is the beginning.

- f. Ms. Maiden is curious about the moment when the ball hits the ground. Which equation below would help her answer the question? Explain how you know.

$$0 = -16(x - 2)^2 + 64$$

$$y = -16(0 - 2)^2 + 64$$

Because $y=0$ is the ground

2. Solving

- a. You haven't learned how to solve equations like $0 = -16(x-2)^2 + 64$ yet, (although you should be able to evaluate $y = -16(0-2)^2 + 64$). The process for solving $0 = -16(x-2)^2 + 64$ is very similar to how we have solved equations all year. First solve these equations for x .

$$x + 3 = 5$$

$$\begin{array}{r} -3 \quad -3 \\ \hline x = 2 \end{array}$$

$$3x - 5 = 7$$

$$\begin{array}{r} +5 \quad +5 \\ \hline 3x = 12 \\ \hline \frac{3x}{3} = \frac{12}{3} \\ \hline x = 4 \end{array}$$

$$.5x + 2 = -1$$

$$\begin{array}{r} -2 \quad -2 \\ \hline .5x = -3 \\ \hline \frac{.5x}{-.5} = \frac{-3}{-.5} \\ \hline x = -6 \end{array}$$

$$\frac{2}{3}x - \frac{1}{3} = \frac{1}{3}$$

$$\begin{array}{r} +\frac{1}{3} \quad +\frac{1}{3} \\ \hline \frac{2}{3}x = \frac{2}{3} \\ \hline \frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{\frac{2}{3}}{\frac{2}{3}} \\ \hline x = 1 \end{array}$$

- b. Equations with parentheses can also be solved without using the distributive property. Here's an example:

$4(x-1) = 20$ $\div 4 \quad \div 4$	$x-1 = 5$ $+1 \quad +1$	$x = 6$	$4(6-1) = 20$ $4(5) = 20$
1. Divide by 4	2. Add 1	3. Solution	4. Check solution

Use the example problem to solve the following equations for x .

$$3(x+4) = 12$$

$$\begin{array}{r} \frac{3(x+4) = 12}{3} \\ \hline x+4 = 4 \\ -4 \quad -4 \\ \hline x = 0 \end{array}$$

$$-2(x-5) = 16$$

$$\begin{array}{r} \frac{-2(x-5) = 16}{-2 \quad -2} \\ \hline x-5 = -8 \\ +5 \quad +5 \\ \hline x = -3 \end{array}$$

$$.25(x+7) = -1$$

$$\begin{array}{r} \frac{.25(x+7) = -1}{.25} \\ \hline x+7 = -4 \\ -7 \quad -7 \\ \hline x = -11 \end{array}$$

$$\frac{2}{3}(x-1) = \frac{1}{3}$$

$$\begin{array}{r} \frac{\frac{2}{3}(x-1) = \frac{1}{3}}{\frac{2}{3}} \\ \hline x-1 = \frac{1}{2} \\ +1 \quad +1 \\ \hline x = \frac{3}{2} \end{array}$$

- c. Equations with an x^2 term can be solved by taking the square root. Solve each equation below

$$x^2 = 16$$

$$x^2 = 36$$

$$x^2 = 100$$

$$x^2 = 1$$

$$x^2 = 0$$

$$x = \pm 4 \quad x = \pm 6 \quad x = \pm 10 \quad x = \pm 1 \quad x = 0$$

- d. Putting it together: You are now ready to solve quadratic equations like the football equation. Here's an example:

$4(x-1)^2 + 1 = 17$ $-1 \quad -1$	$4(x-1)^2 = 16$ $\div 4 \quad \div 4$	$(x-1)^2 = 4$ $\sqrt{\quad} \quad \sqrt{\quad}$	$x-1 = 2 \text{ or } x-1 = -2$ $2 \cdot 2 = 4 \text{ and } -2 \cdot -2 = 4$	$x = 3 \text{ or } x = -1$
1. Subtract 1	2. Divide 4	3. Square Root	4. Add 1	5. Solution

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3. Back to Context:

a. Solve the equation $0 = -16(x-2)^2 + 64$ for x .

$$\begin{aligned}
 & \frac{-64}{-16} = \frac{-64}{-16} \\
 & -64 = -16(x-2)^2 \\
 & \frac{-64}{-16} = \frac{-16(x-2)^2}{-16} \\
 & \sqrt{4} = \sqrt{(x-2)^2} \\
 & \pm 2 = x-2
 \end{aligned}$$

$x-2=2$
 $x=4$
 $x-2=-2$
 $x=0$

b. What do your solutions tell you about the football?

It started on the ground ($x=0$) & hit the ground 4 seconds later ($x=4$).

c. Evaluate $y = -16(0-2)^2 + 64$.

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$$\begin{aligned}
 y &= -16(4)^2 + 64 \rightarrow -16(0-2)^2 + 64 \\
 &= -16(16) + 64 = -256 + 64 = -16(4) + 64 = 0
 \end{aligned}$$

d. What does your value in part c tell you about the football?

Ball started on the ground

4. Return to the football equation $y = -16(x-4)^2 + 64$ and sketch a more detailed graph. Use your answers from question 3 to label the graph.



