## Example: Calculating the Standard Deviation

A consumers' organization is planning a study of the various brands of batteries that are available. As part of its planning, it measures lifetime (i.e., how long a battery can be used before it must be replaced) for each of six batteries of Brand A and eight batteries of Brand B. The dot plots below show the battery lives for each brand:


1. Does one brand of battery tend to last longer, or are they roughly the same? What calculations could you do in order to compare the battery lives of the two brands?
2. Do the battery lives tend to differ more from battery to battery for Brand A or for Brand B?
3. How do you measure variability of these data sets? One way is by calculating standard deviation. We will use Brand A to demonstrate the steps:

- First, find the mean. In this case, the mean is $\mathbf{1 0 1 .}$
- Second, find each deviation from the mean (this means, take each data point and subtract the mean from it).
- Third, square the deviations from the mean. For example, when the deviation from the mean is -18 , the squared deviation from the mean is $(-18)^{2}=324$.

| Brand A: Life (Hours) | 83 | 94 | 96 | 106 | 113 | 114 |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Deviation from the Mean | $83-101=\mathbf{- 1 8}$ | $94-101=\mathbf{- 7}$ | $96-101=\mathbf{- 5}$ | $106-101=\mathbf{5}$ | $113-101=\mathbf{1 2}$ | $114-101=13$ |
| Squared Deviations from <br> the Mean | 324 | 49 | 25 | 25 | 144 | 169 |

- Then, add up the squared deviations:

$$
324+49+25+25+144+169=736
$$

This result is the sum of the squared deviations.
The number of values in the data set is denoted by $\boldsymbol{n}$. In this example, $\boldsymbol{n}$ is 6 .

- Then, you divide the sum of the squared deviations by $n-1$, which here is $6-1$, or 5 .

$$
\frac{736}{5}=147.2
$$

- Finally, you take the square root of 147.2 , which to the nearest hundredth is 12.13 .

That is the standard deviation! It seems like a very complicated process at first, but we will use the TI-84 calculator to do this for us later.

So what does this crazy calculation tell us? It tells us that $68 \%$ of the data (the majority of the data) is within 12.13 hours of the mean (on both sides of the mean).

So, for Brand A, $68 \%$ of the batteries have lifetimes in between 88.87 and 113.13 hours. ( $101-12.13$, and $101+12.13$ )

## Do you think the standard deviation will be higher or lower than this for Battery Brand B ?

Now you can calculate the standard deviation of the lifetimes for the eight Brand B batteries. The mean was 100.5. We already have calculated some of the deviations from the mean. Fill in the rest of the deviations.

| Brand B: Life (Hours) | 73 | 76 | 92 | 94 | 110 | 117 | 118 | 124 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Deviation from the Mean | -27.5 |  | -8.5 |  | 9.5 |  | 17.5 | 23.5 |
| Squared Deviation from the Mean |  |  |  |  |  |  |  |  |

1. Write the squared deviations in the table above.
2. Add up the squared deviations in the third row. What sum do you get? $\qquad$
3. What is the value of $n$ for this data set? $\qquad$ Divide the sum of the squared deviations by $n-1$. Record your answer to the nearest hundredth here $\qquad$
4. Take the square root to find the standard deviation. Record your answer to the nearest hundredth here $\qquad$
5. How would you interpret the standard deviation that you found in Exercise 4? (Remember to give your answer in the context of this question. Interpret your answer to the nearest hundredth.)
6. Between what two numbers will $68 \%$ of the data fall?

Using Technology to find standard deviation:
TI-84 Calculator:

- https://www.youtube.com/watch?v=Qr8jdhkPWLI (Note Standard Deviation is $S_{x}$, Mean is $\bar{x}$ )


## Exercises

Jenna has bought a new hybrid car. Each week for a period of seven weeks, she has noted the fuel efficiency (in miles per gallon) of her car. The results are shown below.

$$
\begin{array}{lllllll}
45 & 44 & 43 & 44 & 45 & 44 & 43
\end{array}
$$

1. Use the TI-84 Calculator to calculate the standard deviation of these results to the nearest hundredth.
2. What is the meaning of the standard deviation you found in Question 6?
3. Sam also has been keeping track of the fuel efficiency of his car. He calculated the standard deviation to be 3.15. Which car has a more reliable fuel efficiency? What factors could be affecting each person's standard deviation?
