

I. Order of Operations (P, E, M&D, A&S)

- Parentheses and other grouping symbols.
- Exponential expressions.
- Multiplication, Division, & taking the Opposite.
- Addition & Subtraction.

Simplify each numerical expression. Show all work! Only use a calculator to check.

1) $6 + 2 \times 8 - 12 + 9 \div 3$

$$\begin{aligned} 6 + 16 - 12 + 3 \\ 22 - 12 + 3 \\ 10 + 3 \\ 13 \end{aligned}$$

2) $25 - (2^3 + 5 \times 2 - 3)$

$$\begin{aligned} 25 - (8 + 10 - 3) \\ 25 - 15 \\ 10 \end{aligned}$$

3) $\frac{-2 \cdot (-30) + 0.5 \cdot 20}{4^2 - 6}$

$$\frac{-60 + 10}{16 - 6} = \frac{-50}{10} = -5$$

4) $\frac{15 - [8 - (2 + 5)]}{18 - 5^2}$

$$\frac{15 - (8 - 7)}{18 - 25} = \frac{15 - 1}{-7} = \frac{14}{-7} = -2$$

II. Evaluating Algebraic Expressions

To evaluate an algebraic expression:

- Substitute the given value(s) of the variable(s).
- Use order of operations to find the value of the resulting numerical expression.

Evaluate.

1) $x \left(\frac{y}{2} + 3z^2 \right) - 2x$ if $x = \frac{1}{2}$, $y = 4$, $z = -2$

$$\begin{aligned} \frac{1}{2} \left(\frac{4}{2} + 3(-2)^2 \right) - 2 \left(\frac{1}{2} \right) \\ \frac{1}{2} (2 + 3 \cdot 4) - 1 \\ \frac{1}{2} (2 + 12) - 1 = \frac{1}{2} (14) - 1 \\ 7 - 1 = 6 \end{aligned}$$

2) $12a - 4a^2 + 7a^3$ if $a = -3$

$$\begin{aligned} 12(-3) - 4(-3)^2 + 7(-3)^3 \\ -36 - 4 \cdot 9 + 7 \cdot (-27) \\ -36 - 36 - 189 = -261 \end{aligned}$$

$$3) \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ if } a=1, b=-4, c=-21$$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-21)}}{2(1)} = \frac{4 \pm \sqrt{16 + 84}}{2}$$

$$\frac{4 \pm \sqrt{100}}{2} = \frac{4 \pm 10}{2} = \frac{14}{2} = 7.$$

$$5) \frac{3(x+y) - 2(x-y)}{5x+y} \text{ if } x=3 \text{ and } y=4$$

$$\frac{3(3+4) - 2(3-4)}{5(3)+4}$$

$$\frac{3(7) - 2(-1)}{15+4} = \frac{21+2}{19} = \frac{23}{19}$$

$$7) A = P \left(1 + \frac{r}{n}\right)^{nt} \text{ if } P=650, r=6\%, n=2, t=15$$

$$A = 650 \left(1 + \frac{.06}{2}\right)^{2 \cdot 15}$$

$$= 650(1.03)^{30} \approx 1577.72$$

$$4) 1.2(3)^x \text{ if } x=3$$

$$1.2 \cdot 3^3 = 1.2 \cdot 27$$

$$32.4$$

$$6) 2\left(\frac{1}{3}\right)^x \text{ if } x=2$$

$$2\left(\frac{1}{3}\right)^2 = 2 \cdot \left(\frac{1}{9}\right) = \frac{2}{9}$$

$$8) \text{ If } k \odot n = k^3 - 3n,$$

then evaluate $7 \odot 5$

$$7 \odot 5 = 7^3 - 3 \cdot 5$$

$$343 - 15 = 328$$

III. Properties of Exponents - Complete the example problems.

PROPERTY		EXAMPLE
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$x^4 \cdot x^2 =$
Power of a Power	$(a^m)^n = a^{m \cdot n}$	$(x^4)^2 =$
Power of a Product	$(ab)^m = a^m b^m$	$(2x)^3 =$
Negative Power	$a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$	$x^{-3} =$
Zero Power	$a^0 = 1 \quad (a \neq 0)$	$4^0 =$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$	$\frac{x^3}{x^2} =$
Power of Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (b \neq 0)$	$\left(\frac{x}{y}\right)^3 =$

Simplify each expression. Answers should be written using positive exponents.

$$1) g^5 \cdot g^{11} = \frac{g^{16}}{g^0}$$

$$3) w^{-7} = \frac{1}{w^7}$$

$$5) (3x^7)(-5x^{-3}) = \frac{-15x^4}{1}$$

$$7) \frac{-15x^2y^{-2}}{25x^3y^3} = \frac{-3x^{-1}}{5y^5}$$

$$2) (b^5)^3 = \frac{b^{15}}{1}$$

$$4) \frac{y^{12}}{y^8} = y^4$$

$$6) (-4a^{-5}b^0c)^2 = \frac{16a^{-10}c^2}{1}$$

$$8) \left(\frac{4x^9}{12x^4}\right)^3 = \left(\frac{1x^5}{3}\right)^3 = \frac{x^{15}}{27}$$

IV. Solving Linear Equations

To solve linear equations, first simplify both sides of the equation. If the equation contains fractions, multiply the equation by the LCD to clear the equation of fractions. Use the addition and subtraction properties of equality to get variables on one side and constants on the other side of the equal sign. Use the multiplication and division properties of equality to solve for the variable. Express all answers as fractions in lowest terms.

Examples:

$$a) 3(x+5) + 4(x+2) = 21$$

$$3x + 15 + 4x + 8 = 21$$

$$7x + 23 = 21$$

$$7x = -2$$

$$x = -\frac{2}{7}$$

$$b) 2(5x-4) - 10x = 6x + 3(2x-5)$$

$$10x - 8 - 10x = 6x + 6x - 15$$

$$-8 = 12x - 15$$

$$7 = 12x$$

$$\frac{7}{12} = x$$

$$c) \frac{2}{3}x + 5 = 6x - \frac{3}{4}$$

$$12\left(\frac{2}{3}x + 5 = 6x - \frac{3}{4}\right)$$

$$8x + 60 = 72x - 9$$

$$69 = 64x$$

$$\frac{69}{64} = x$$

Solve for the indicated variable:

$$1) 3n + 1 = 7n - 5$$

$$-3n \quad -3n$$

$$1 = 4n - 6$$

$$+5 \quad +5$$

$$6 = 4n$$

$$n = \frac{3}{2}$$

$$2) 2[x + 3(x-1)] = 18$$

$$2(x + 3x - 3) = 18$$

$$2(4x - 3) = 18$$

$$4x - 3 = 9$$

$$+3 \quad +3$$

$$4x = 12$$

$$x = 3$$

$$3) 6(y+2) - 4 = -10$$

$$6y + 12 - 4 = -10$$

$$6y + 8 = -10$$

$$6y = -18$$

$$y = -3$$

$$4) 2x^2 = 50$$

$$x^2 = 25$$

$$x = 5 \text{ OR } x = -5$$

$$5) 5 + 2(k+4) = 5(k-3) + 10$$

$$5 + 2k + 8 = 5k - 15 + 10$$

$$13 + 2k = 5k - 5$$

$$13 = 3k - 5$$

$$18 = 3k$$

$$k = \frac{18}{3} = 6$$

$$6) 6 + 2x(x-3) = 2x^2$$

$$6 + 2x^2 - 6x = 2x^2$$

$$-2x^2 \quad -2x^2$$

$$6 - 6x = 0$$

$$6 = 6x$$

$$x = 1$$

$$7) \frac{2}{3}x - 18 = \frac{x}{6}$$

$$-18 = \frac{x}{6} - \frac{22}{3}x$$

$$-18 = -\frac{3}{6}x = -\frac{1}{2}x$$

$$x = 36$$

$$8) \frac{x-2}{3} = \frac{2x+1}{4}$$

$$4(x-2) = (2x+1) \cdot 3$$

$$4x - 8 = 6x + 3$$

$$-8 = 2x + 3$$

$$-11 = 2x \rightarrow x = -\frac{11}{2}$$

V. Operations With Polynomials

To add or subtract polynomials, just combine like terms.

To multiply polynomials, multiply the numerical coefficients and apply the rules for exponents.

Examples:

a) $(x^2 + 3x - 2) - (3x^2 - x + 5)$
 $x^2 + 3x - 2 - 3x^2 + x - 5$
 $-2x^2 + 4x - 7$

b) $3x(2x + 5)^2$
 $3x(4x^2 + 20x + 25)$
 $12x^3 + 60x^2 + 75x$

c) $4(5x^2 + 3x - 4) + 3(-2x^2 - 2x + 3)$
 $20x^2 + 12x - 16 - 6x^2 - 6x + 9$
 $14x^2 + 6x - 7$

d) $(4x - 5)(3x + 7)$
 $12x^2 + 28x - 15x - 35$
 $12x^2 + 13x - 35$

Perform the indicated operations and simplify:

1) $(7x^2 + 4x - 3) - (-5x^2 - 3x + 2)$
 $12x^2 + 7x - 5$

2) $(7x - 3)(3x + 7)$
 $21x^2 + 40x - 21$

3) $(4x + 5)(5x + 4)$
 $20x^2 + 41x + 20$

4) $(n^2 + 5n + 3) + (2n^2 + 8n + 8)$
 $3n^2 + 13n + 11$

5) $(5x^2 - 4) - 2(3x^2 + 8x + 4)$
 $5x^2 - 4 - 6x^2 - 16x - 8$
 $-1x^2 - 16x - 12$

6) $-2x(5x + 11)$
 $-10x^2 - 22x$

7) $(2m + 6)(2m + 6)$
 $4m^2 + 24m + 36$

8) $(5x - 6)^2$
 $25x^2 - 60x + 36$

VI. Factoring Polynomials

Examples:

Factoring out the GCF

a) $6x^2 + 21x$

$$3x(2x + 7)$$

Difference of Squares

b) $x^2 - 64$

$$(x - 8)(x + 8)$$

Perfect Square Trinomial

c) $x^2 - 10x + 25$

$$(x - 5)^2$$

Trinomial
d) $3x^2 + 7x + 2$

$(3x + 1)(x + 2)$

Trinomial
e) $2x^2 - 13x + 15$

$(2x - 3)(x - 5)$

Trinomial
f) $6x^2 + x - 1$

$(3x - 1)(2x + 1)$

Factor Completely.

1) $16y^2 + 8y$

$8y(2y + 1)$

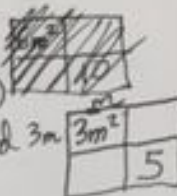
2) $18x^2 - 12x$

$6x(3x - 2)$

3) $6m^2 - 60m + 10$

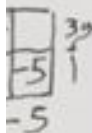
$2(3m^2 - 30m + 5)$

Cannot be factored further



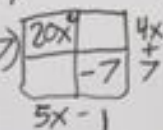
4) $6y^2 - 13y - 5$

$(3y + 1)(2y - 5)$



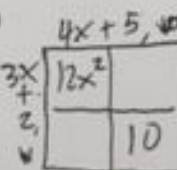
5) $20x^2 + 31x - 7$

$(5x - 1)(4x + 7)$



6) $12x^2 + 23x + 10$

$(4x + 5)(3x + 2)$

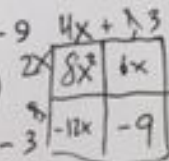


7) $x^2 - 2x - 63$

$(x - 9)(x + 7)$

8) $8x^2 - 6x - 9$

$(4x + 3)(2x - 3)$



9) $x^2 - 121$

$(x + 11)(x - 11)$

VII. Linear Equations in Two Variables

Examples:

a) Find the slope of the line passing through the points $(-1, 2)$ and $(3, 5)$.

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{5 - 2}{3 - (-1)} = \frac{3}{4}$$

b) Write the equation of the line with a slope of 3 and passing through the point $(2, -1)$

$y = mx + b$

$-1 = 3(2) + b$

$-7 = b$

Equation: $y = 3x - 7$