CCSS Advanced Algebra Reversing Operations (Other Functions) Part 2

Complete work in your math notebook.

1. Absolute Value Equations
   a. Evaluate or solve each of the following:
   - $|4| = ?$ 4
   - $|-4| = ?$ 4
   - $|x+3| = ?$ when $x = -10$ $|-10+3| = |-7| = 7$
   - $|x| = 1$, $x = ?$ [2 answers] $x=1$, $x=-1$
   - $|x| = -2$, $x = ?$ No solution
   - $|x-1| = 0$, $x = ?$ $x=1$

   2 #s are 1 step from zero

   No #s are negative distance from zero

   1 # is zero distance from zero

   b. What does it mean to take the Absolute Value of a number, for example, $|-32| = 32$

   Find distance from zero

   c. To solve equations with Absolute Value, you need to understand that $|5| = |-5| = 5$. Watch the screencast on Solving Absolute Value Equations. Then practice on the problems below.

   i. $|x-4| = 2$
   ii. $|x+5| - 3 = 10$
   iii. $2|x-1| + 4 = 10$
   iv. $-4|x+1| + 7 = -13$
   v. $-4|x+1| + 7 = 7$
   vi. $-4|x+1| + 7 = 27$

   i. $|x-4|=2$; $x-4=2$, $x=6$; $x-4=-2$, $x=2$

   ii. $|x+5|-3=10$; $|x+5|=13$; $x+5=13$, $x=8$; $x+5=-13$, $x=-18$

   iii. $2|x-1|+4=10$; $2|x-1|=6$; $|x-1|=3$; $x-1=3$, $x=4$; $x-1=-3$, $x=-2$

   iv. $-4|x+1|+7=-13$; $-4|x+1|=-20$; $|x+1|=5$; $x+1=5$, $x=4$; $x+1=-5$, $x=-6$

   v. $-4|x+1|+7=7$; $-4|x+1|=0$; $|x+1|=0$; $x+1=0$; $x=-1$

   vi. $-4|x+1|+7=27$; $-4|x+1|=20$; $\frac{-4|x+1|}{-4}=\frac{20}{-4}$; $|x+1|=-5$

   No Solution

2. Rational Equations (Equations involving fractions)
   a. Solve each equation below by first removing the fractions.

   i. $\frac{x}{3} + 1 = \frac{5}{2}$
   ii. $\frac{2x}{5} + \frac{1}{20} = \frac{x}{10}$
   iii. $\frac{(x-4)^2}{2} + 1 = \frac{11}{2}$
   iv. $\frac{2|x-1|}{3} - \frac{1}{6} = 2$
   v. $(\frac{2}{3}x + 1)(\frac{x}{5} - \frac{1}{10}) = 0$

   i. $3\cdot(\frac{x}{3}+1=\frac{5}{2})\cdot 3$; $2\cdot(x+3=\frac{15}{2})\cdot 2$; $2x+6=15$; $2x=9$; $x=9/2$

   ii. $20(\frac{2x}{5}+\frac{1}{20}=\frac{x}{10})\cdot 20$; $8x+1=2x$; $1=-6x$; $-1/6=x$

   iii. $2\cdot(\frac{(x-4)^2}{2}+1=\frac{11}{2})\,2$; $(x-4)^2+2=11$; $(x-4)^2=9$; $x-4=3$, $x=7$; $x-4=-3$, $x=1$

   b. Rational Equations can also have the variables in the denominator of the fraction. Consider the equation $\frac{5}{x} + 3 = \frac{2}{x}$. What operation would remove the fractions in this problem?

   Multiply by "x"

   c. Simplify the equation $\frac{5}{x} + 3 = \frac{2}{x}$ by removing the fractions and then solve.

   $x\cdot(\frac{5}{x}+3=\frac{2}{x})\cdot x \rightarrow 5+3x=2 \rightarrow 3x=-3 \rightarrow x=-1$

3. More Rational Equations:

a. $5x \cdot \left(\frac{1}{x} + \frac{6}{5x} = 1\right) \cdot 5x$

$5 + 6 = 5x$

$11 = 5x$

$\frac{11}{5} = x$

b. $2x^2 \left(\frac{1}{x^2} + \frac{1}{x} = \frac{1}{2x^2}\right) \cdot 2x^2$

$2 + 2x = 1$

$2x = -1$

$x = -1/2$

c. $x \cdot \left(x + 1 = \frac{72}{x}\right) \cdot x$

$x^2 + x = 72$

$(-9)^2 + (-9) = 72$ $x = -9$

$8^2 + 8 = 72$ $x = 8$

d. $x-3 \left(x + \frac{x-1}{x-3} = \frac{2}{x-3}\right) \cdot x-3$

$x^2 - 3x + x - 1 = 2$

$x^2 - 2x = 3$

$3^2 - 2(3) = 3$

$(-1)^2 - 2(-1) = 3$

$x = 3$ $x = -1$

4. Consider the equation $\frac{1}{x-1} + \frac{1}{x} = \frac{-1}{x(x-1)}$.

a. What would you need to multiply the equation by to remove the fractions?

b. Explain, based on your answer to part (a), why the equation above can be changed to $x + x - 1 = -1$.

c. Solve this equation for x.

d. Check your solution by plugging the value of x into the original equation. What happened? This is called an extraneous solution.

e. Show that $\frac{1}{x-2} + \frac{1}{x+2} = \frac{4}{(x-2)(x+2)}$ has an extraneous solution.

f. Use desmos.com or the TI-84 to check the solutions in parts (c) and (e). What is it about the graphs that creates an extraneous solution? Explain fully.

a) By $x(x-1)$

b) $x(x-1)\left(\frac{1}{x-1} + \frac{1}{x} = \frac{-1}{x(x-1)}\right)x \cdot (x-1) \rightarrow x + x - 1 = -1$

Denominators cancel

c) $x + x - 1 = -1$

$2x - 1 = -1$

$2x = 0$

$x = 0$

d) $\frac{1}{0-1} + \frac{1}{0} = \frac{-1}{0(0-1)}$ Error, Can't $\div$ by 0.

e) $(x-2)(x+2)\left(\frac{1}{x-2} + \frac{1}{x+2} = \frac{4}{(x-2)(x+2)}\right)(x-2)(x+2) \longrightarrow x + 2 + x - 2 = 4$

$2x = 4$

$x = 2$ Makes denominator $= 0$