Name: KEY

1. Consider the system of equations $x^2 + y^2 = 100$ and $y = x^2 - 28$.

   a. Find the solution(s) to the system, and give your answers as ordered pairs.

$$x^2 + y^2 = 100, \quad y = x^2 - 28$$
$$y + 28 = x^2$$
$$y + 28 + y^2 = 100$$
$$\;\;\;_{-100}\qquad {}_{-100}$$
$$y^2 + y - 72 = 0$$

$$(y+9)(y-8) = 0$$
$$y = -9, \quad y = 8$$
Solve for x:
$$-9 = x^2 - 28 \qquad 8 = x^2 - 28$$
$$19 = x^2 \qquad\quad 36 = x^2$$
$$\pm\sqrt{19} = x \qquad \pm 6 = x$$

4 solutions.
$(\sqrt{19}, -9)$
$(-\sqrt{19}, -9)$
$(6, 8)$
$(-6, 8)$

   b. Modify the system of equations so there is only 1 solution. What is the solution to your system?

Lots of answers.
$$x^2 + y^2 = 100$$
$$y = x^2 + 10$$
OR
$$x^2 + y^2 = 100$$
$$y = -x^2 - 10$$

   c. Modify the system of equations so there are no solutions. Describe how you know your system will have no solutions.

Lots of answers
$$x^2 + y^2 = 1$$
$$y = x^2 - 28$$
circle is smaller than parabola
OR
$$x^2 + y^2 = 100$$
$$y = x^2 + 12$$
circle is below parabola

2. Consider the system of equations $y = 4\sqrt{x+5} - 2$ and $y = 2x + 2$

   a. Find the solution(s) to the system. Give your answers as ordered pairs.

$$4\sqrt{x+5} - 2 = 2x + 2$$
$$4\sqrt{x+5} = 2x + 4$$
$$\sqrt{x+5} = \tfrac{1}{2}x + 1$$
$$x + 5 = (\tfrac{1}{2}x + 1)^2$$
$$x + 5 = \tfrac{1}{4}x^2 + x + 1$$

$$\left(\tfrac{1}{2}x + 1\right)^2$$
$$\tfrac{1}{4}x^2 + \tfrac{1}{2}x + \tfrac{1}{2}x + 1$$
$$\tfrac{1}{4}x^2 + x + 1$$

$$0 = \tfrac{1}{4}x^2 - 4$$
$$4 = \tfrac{1}{4}x^2$$
$$16 = x^2$$
$$\pm 4 = x$$
$$y = 2 \cdot 4 + 2 = 8 + 2 = 10$$
$$y = 2 \cdot -4 + 2 = -8 + 2 = -6$$

$(4, 10)$ & $(-4, -6)$
But $(-4, -6)$ is an extraneous solution, because $-6 \neq 4\sqrt{-4+5} - 2$
$-6 \neq 4\sqrt{1} - 2$

   b. What do your solution(s) tell you about the graph of the system?

The graphs cross at $(4, 10)$. They would cross at $(-4, -6)$ if we drew the other arm of the square root.

3. You are shooting a laser at a clay projectile that is flying through the air. Your goal is for your laser (which travels in a straight line) to hit the projectile at its highest point. The way you aim your laser is by telling it the linear equation of the path of the laser. The projectile can be modeled by the function $h(t) = -16t^2 + 128t$, where "h(t)" is the height in feet, and "t" is the time in seconds.

a. Draw a sketch of the situation. Label any important points.

$$-16t^2 + 128t = 0$$
$$-16t(t-8) = 0$$
$$t=0, \quad t=8.$$
$$h = \frac{0+8}{2} = 4$$
$$K = -16 \cdot 4(4-8) = 256$$
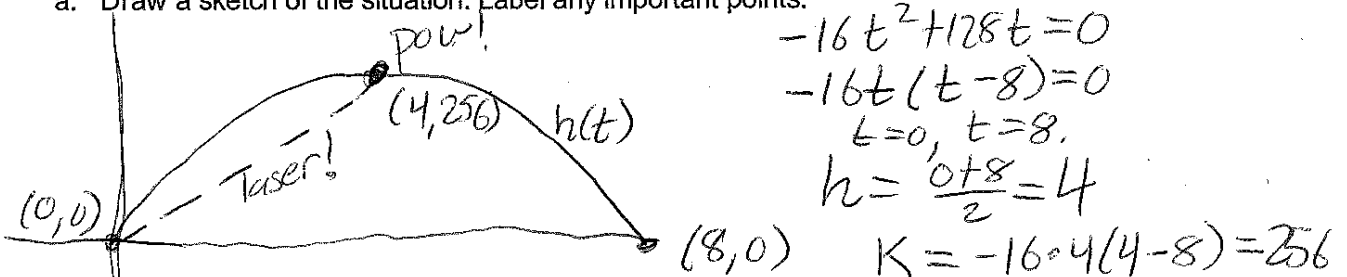
b. What linear equation should you program into your laser so you hit the projectile at its highest point?

$$y = \frac{256}{4}x = y = 64x.$$

4. You have been offered the choice of two allowance plans. The first plan pays 1 cent on the first week, 2 cents the second week, 4 cents the third week, and continues doubling every week. The second plan pays you $100,000 every week.

a. Write a system of equations to model the two allowance plans.

Plan 1: $y = .01(2)^{x-1}$ Plan 2: $y = 100,000 \cdot x$

b. The second plan is clearly better at first. Complete the tables below to show how much each plan earns for the given weeks.

Week	Plan 1	Plan 2
1	.01	100,000
2	.02	200,000
3	.04	500,000
10		1,000,000
100		10,000,000
1000		100,000,000

b. After how many weeks is plan 1 better than plan 2?

5. You are selling 12 balloons and 8 stuffed animals for $60. If you reduce the price of a balloon by $1 and double the price of a stuffed animal, you will make $72. $X = balloon, y = stuffed$ animal

a. What is the original price of a stuffed animal?

$$12x + 8y = 60$$
$$12(x-1) + 8(2y) = 72$$
$$12x - 12 + 16y = 72, \quad 12x + 16y = 84$$

$$12x + 8y = 60$$
$$-(12x + 16y = 84)$$
$$\overline{-8y = -24}$$
$$\boxed{y = 3}$$

b. What is the original price of a balloon?

$$12x + 8 \cdot 3 = 60$$
$$12x + 24 = 60$$
$$12x = 36, \quad x = 3$$

c. If balloons cost you $3 to purchase and stuffed animals cost you $5, how much profit will you make at the NEW price if you sell all your inventory?

12 balloons, 8 stuffed animal

Cost: $12 \cdot 3 + 8 \cdot 5 = 36 + 40 = 76$

Earn: 72. Profit = 72 - 76 = -4

You will lose 4 dollars.

6. Two new indoor skydiving companies are opening in SE Portland. They advertise their prices based on the number of minutes you will be in the air. Here are their prices:

Minutes of Airtime	Hawthorn Air-Hop	Division Sky-Dive
10	$73	$20
20	$128.50	$82.50
50	$295	$270
100	$572.50	$582.50

a. Write a system of linear equations that models Hawthorn Air-Hop's and Division Sky-Dive's prices.

H
$$m = \frac{128.50 - 73}{20 - 10} = \frac{55.50}{10} = 5.55$$
$$y = 5.55x + b.$$
$$73 = 5.55 \cdot 10 + b$$
$$73 = 55.5 + b$$

D
$$m = \frac{82.50 - 20}{20 - 10} = \frac{62.50}{10} = 6.25$$
$$y = 6.25x + b$$
$$20 = 6.25(10) + b$$
$$20 = 62.50 + b$$

b. Solve your system of equations. What does your solution mean in the context of the problem?
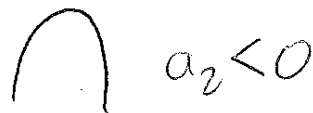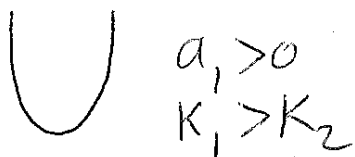
H
$$y = 5.55x + 17.5$$

D
$$y = 6.25x - 42.50$$
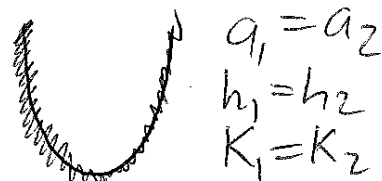
$$5.55x + 17.5 = 6.25x - 42.50$$
$$60 = .7x$$
$$85.71 = x$$

If you skydive for 85.71 minutes, the two will cost the same.

7. Consider a system of 2 parabolas. Draw as many different types of systems of 2 parabolas as you can, and describe how "a," "h," and "k," are related for each equation in the system. Be sure to include a system with 0 solutions and a system with infinite solutions.
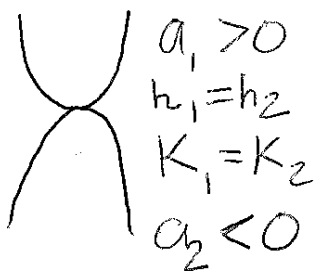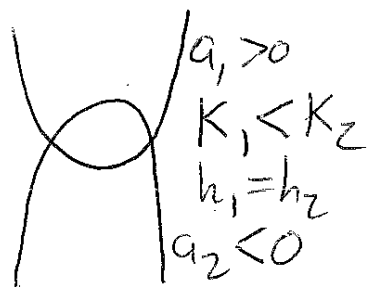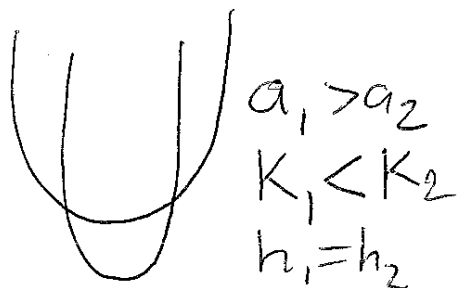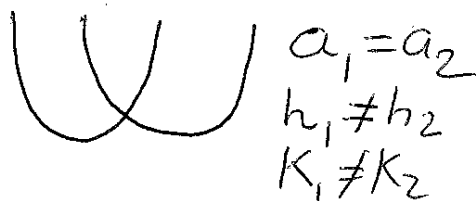
$a_1 > 0$
$K_1 > K_2$

$a_2 < 0$

0 solutions

$a_1 = a_2$
$h_1 = h_2$
$K_1 = K_2$

2 identical parabolas

Infinite Solutions

$a_1 > 0$
$h_1 = h_2$
$K_1 = K_2$
$a_2 < 0$

1 Solution

$a_1 > a_2 > 0$
$h_1 = h_2$
$K_1 = K_2$

$a_1 = a_2$
$h_1 \neq h_2$
$K_1 \neq K_2$

$a_1 > a_2$
$K_1 < K_2$
$h_1 = h_2$

$a_1 > 0$
$K_1 < K_2$
$h_1 = h_2$
$a_2 < 0$

2 solutions