

Analyzing and Solving Polynomial Equations

State the number of complex roots, the possible number of real and imaginary roots, the possible number of positive and negative roots, and the possible rational roots for each equation. Then find all roots.

1) $x^4 - 5x^2 - 36 = 0$

$(x^2 - 9)(x^2 + 4)$

$(x+3)(x-3)(x^2+4)$

real roots: $x = -3, 3$

Cpx Roots: $x = 2i, -2i$

3) $x^3 - 2x^2 + 3x - 6 = 0$ 1, 2, 3, 6

~~1 is not a root~~ 2 is root $x_2 + 3$

$$\begin{array}{r} x+2 \overline{) x^3 - 2x^2 + 3x - 6} \\ \underline{-(x^3 - 2x^2)} \\ 3x - 6 \\ \underline{-(3x - 6)} \\ 0 \end{array}$$

$(x-2)(x^2+3)$
 $x = 2, x = \pm\sqrt{3}i$

5) $x^4 + 6x^2 + 8 = 0$

$(x^2 + 4)(x^2 + 2)$

$x = \pm 2i, \pm \sqrt{2}i$

2) $x^3 + 3x^2 - 14x - 20 = 0$ $-20 = 1 \cdot 20$
 $2 \cdot 10$
 $4 \cdot 5$

~~5 is a root~~

$$\begin{array}{r} x+5 \overline{) x^3 + 3x^2 - 14x - 20} \\ \underline{-(x^3 + 5x^2)} \\ -2x^2 - 14x - 20 \\ \underline{-(-2x^2 - 10x)} \\ -4x - 20 \\ \underline{-(-4x - 20)} \\ 0 \end{array}$$

$(x+5)(x^2 - 2x - 4)$
 $x = \frac{2 \pm \sqrt{4 - 4(-4)(1)}}{2}$
 $x = \frac{2 \pm \sqrt{4 + 16}}{2}$
 $x = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$
 roots

4) $x^4 - 14x^2 + 45 = 0$

$(x^2 - 9)(x^2 - 5)$
 $(x+3)(x-3)(x+\sqrt{5})(x-\sqrt{5})$
 $x = -3, 3, -\sqrt{5}, \sqrt{5}$

6) $x^4 + 3x^2 - 18 = 0$

$(x^2 + 6)(x^2 - 3)$
 $x = \pm\sqrt{6}i, x = \pm\sqrt{3}$

7) $x^3 - 1 = 0$ ← Same as 3rd roots of unity.

$(x-1)(x^2+x+1)$
 $x = 1, x = \frac{-1 \pm \sqrt{1-4}}{2}$
 $x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

8) $x^3 + 3x^2 - x - 3 = 0$

$x^2(x+3) - 1(x+3)$
 $(x^2 - 1)(x+3)$
 $(x+1)(x-1)(x+3)$
 $x = -1, x = 1, x = -3$

$$9) x^3 - 2x^2 - 3x + 6 = 0$$

$$x^2(x-2) - 3(x-2)$$

$$(x^2 - 3)(x-2)$$

$$x = \pm\sqrt{3}, x = 2$$

$$10) x^6 - 2x^4 - 4x^2 + 8 = 0$$

$$x^4(x^2-2) - 4(x^2-2)$$

$$(x^4 - 4)(x^2 - 2)$$

$$(x^2-2)(x^2+2) \quad | \quad x = \pm\sqrt{2}$$

$$x = \pm\sqrt{2}, \quad \swarrow$$

$$x = \pm\sqrt{2}i$$

$$11) x^5 + 2x^4 + 11x^3 + 22x^2 + 24x + 48 = 0$$

$$x^4(x+2) + 11x^2(x+2) + 24(x+2)$$

$$(x+2)(x^4 + 11x^2 + 24)$$

$$(x+2)(x^2+8)(x^2+3)$$

$$x = -2, x = \pm\sqrt{8}i, x = \pm\sqrt{3}i$$

$$12) x^6 + 5x^4 - 4x^2 - 20 = 0$$

$$x^4(x^2+5) - 4(x^2+5)$$

$$(x^4 - 4)(x^2 + 5)$$

$$(x^2-2)(x^2+2)(x^2+5)$$

$$x = \pm\sqrt{2}, x = \pm\sqrt{2}i, x = \pm\sqrt{5}i$$

$$13) x^6 - x^4 - x^2 + 1 = 0$$

$$x^4(x^2-1) - 1(x^2-1)$$

$$(x^4 - 1)(x^2 - 1)$$

$$(x^2+1)(x^2-1)(x^2-1)$$

$$x = \pm i, x = \pm 1$$

$$14) x^8 - 26x^4 + 25 = 0$$

$$(x^4 - 1)(x^4 - 25)$$

$$(x^2+1)(x^2-1)(x^2+5)(x^2-5)$$

$$x = \pm i, x = \pm 1, x = \pm\sqrt{5}i, x = \pm\sqrt{5}$$