

Compound Interest: Compounding Periods

We learned that **compound interest** is where the interest you earn is added into your account so that you earn more money year after year. The extra interest may not seem like much at first, but little bits add up over time to create an exponential curve. In the introduction activity, you calculated the interest earned each year, added it to the principle, then calculated the next year's interest. In reality, banks calculate your interest for a variety of different lengths of time, called the **compounding period**. In the introduction activity, the accounts were **compounded annually** (meaning once per year).

1. **Understanding Compounding Periods**

- a. The table below shows the amount of money in a bank account, with \$1000 initial balance, earning 10% ($=0.10$) annual interest, **compounded annually**.

Complete the table:

Years investment has been in the bank.	Balance at the beginning of the year	Interest earned during the year	Balance at the end of the year.
1	\$1000	$\$1000(0.10) = \100	$\$1000 + \$100 = \$1100$
2	\$1100	$\$1100(0.10) =$	$\$1100 + =$
3			
4			
5			

- b. The table below shows the amount of money in a bank account, with \$1000 initial balance, earning 10% ($=0.10$) annual interest, **compounded semi-annually**. This means you earn interest twice a year. The bank will pay you half of your interest rate at the end of each compounding period.

Complete the table:

Years investment has been in the bank.	Balance at the beginning of the year	Interest earned during the year	Balance at the end of the year.
0.5	\$1000	$\$1000 \cdot 0.05 = \50	$\$1000 + \$50 = \$1050$
1	\$1050	$\$1050 \cdot 0.05 =$	$\$1050 + =$
1.5			
2			
2.5			
3			

- c. In part b, how did you change the annual interest rate to get the semi-annual interest rate?

- d. In part b, how many times per year did you earn interest? (How many compounds were there?)

- e. Do you earn more money **compounding annually** or **compounding semi-annually**? Why?

2. Practicing compounding periods

a. The table below shows the amount of money in a bank account, with \$1000 initial balance, earning 12% ($=0.12$) annual interest, **compounded quarterly**. This means you earn interest four times a year. The bank will pay you _____ of your interest rate at the end of each compounding period.

Complete the table:

Years investment has been in the bank.	Balance at the beginning of the year	Interest earned during the year	Balance at the end of the year.
0.25	\$1000	$\$1000(0.03) = \30	$\$1000 + \$30 = \$1030$
0.5	\$1030	$\$1030(0.03) =$	$\$1030 + =$
0.75			
1			

b. The table below shows the amount of money in a bank account, with \$1000 initial balance, earning 12% ($=0.12$) annual interest, **compounded monthly**. This is the most common compounding period in the real world. This means you earn interest twelve times a year. The bank will pay you _____ of your interest rate at the end of each compounding period.

Complete the table:

Years investment has been in the bank.	Balance at the beginning of the year	Interest earned during the year	Balance at the end of the year.
1/12	\$1000	$\$1000 \cdot 0.01 = \10	$\$1000 + \$10 = \$1010$
2/12	\$1010	$\$1010 \cdot 0.01 =$	$\$1010 + =$
3/12			
4/12			
5/12			
1/2			
7/12			
8/12			
9/12			
10/12			
11/12			
1			

c. Do you earn more money if your account is **compounded quarterly** or **compounded monthly**? Why is this true?

3. An Easier Way

You probably got bored filling out the **compounded monthly** table in problem 2b. Imagine how tedious it would be to fill out a **compounded daily** table. There are even some situations where financial institutions will compound continuously, which basically means every second of every day.

Recall that the annual compound interest formula was $F = P(1 + r)^t$ because you started with P dollars, earned r percent interest every year, and had the account for t years.

The new compound interest formula is similar, except you earn less interest each compounding period and you earn the interest more often than each year. Use the new formula to check the final row of each table.

$$F = P \left(1 + \frac{r}{n} \right)^{n \cdot t}$$