

Comparing Compounds

- You go to the bank and are offered a choice: How many times a year do you want your money to be compounded? You make tables to compare the different compounding periods. Assume you have \$1000 and earn 12% interest.

If you have 12% interest, that becomes 0.12 as a decimal. That is your ANNUAL interest. To change it to your compound interest, divide by "n" (the number of compounds in a year).

Compounded Semi-Annually (n=2) 0.12 ÷ 2 = 0.06				Compounded Quarterly (n=4) 0.12 ÷ 4 = 0.03			
Month	Start	Interest	End	Month	Start	Interest	End
1	1000	—	1000	1	1000	—	1000
2	1000	—	1000	2	—	—	—
3	1000	—	1000	3	1000	30	1030
4	—	—	—	4	—	—	—
5	—	—	—	5	—	—	—
6	1000	60	1060	6	1030	30.9	1060.90
7	1060	—	1060	7	—	—	—
8	1060	—	1060	8	—	—	—
9	—	—	—	9	1060.90	31.83	1092.73
10	—	—	—	10	—	—	—
11	—	—	—	11	—	—	—
12	1060	63.60	1123.60	12	1092.73	32.78	1125.51

Please notice that there are a lot of months where you DO NOT earn interest. This is what the words "semi-annually" and "quarterly" mean. You do not earn interest every month. For semi-annually, you earn interest every 6 months. That is why the majority of months have the number 0 in the interest column. For quarterly, you earn interest every 3 months.

You can verify the final row in the table by using the compound interest formula: $F = P(1 + \frac{r}{n})^{nt}$

$1000(1 + \frac{0.12}{2})^{2 \cdot 1} = 1123.6$	$1000(1 + \frac{0.12}{4})^{4 \cdot 1} = 1125.51$
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2. This time the bank offers you two different plans: you can invest your \$1000 and earn 12% interest, compounded monthly, or you can pay the bank \$1000 today and they will pay you back \$2400 a year, in monthly payments.

Compounded Monthly (n=12)				Monthly Payments (\$2400 total per year)			
Month	Start	Interest	End	Month	Start	Interest	End
1	1000	10	1010	1	-1000	200	-800
2	1010	10.1	1020.1	2	-800	200	-600
3	1020.1	10.2	1030.3	3	-600	200	-400
4	1030.3	10.3	1040.6	4	-400	200	-200
5	1040.6	10.41	1051.01	5	-200	200	0
6	1051.01	10.51	1061.52	6	0	200	200
7	1061.52	10.62	1072.14	7	200	200	400
8	1072.14	10.72	1082.86	8	400		600
9	1082.86	10.83	1093.69	9	600		800
10	1093.69	10.94	1104.62	10	800		1000
11	1104.62	11.05	1115.67	11	1000		1200
12	1115.67	11.16	1126.83	12	1200		1400

a. Which plan is better at the start?

Compounded Monthly

1000 vs -1000

b. Which plan is better after a year?

Monthly Payments

1400 vs 1126.83

c. Use the compound interest formula and a linear equation to verify the final lines of each table.

$$1000 \left(1 + \frac{0.12}{12}\right)^{12} = 1126.83$$

$$y = +200(12) + -1000 = 1400$$

d. We know that exponential plans always beat linear plans, eventually. When does the exponential plan pass the linear plan?

Year 36