

Comparing Linear and Exponential Growth

Recall that **linear** growth means that a constant amount is added repeatedly. The amount that is added is called the **slope**. The amount you start with is called the **y-intercept**. Recall also that **exponential** growth means that a constant amount is multiplied repeatedly. The amount that is multiplied is called the **growth factor**. The amount you start with is called the **y-intercept**.

Linear rule: $y = mx + b$

Exponential rule: $y = b(m)^x$

1. What do the letters "m" and "b" stand for in each model? How are they similar? How are they different?

$m = \text{change}$ Linear = Add
 $b = \text{beginning}$ Exponential = Multiply

2. Mr. Wiggins gives his daughter Celia two choices of payment for raking leaves:

- i. Two dollars for *each* bag of leaves filled, *Ln*
- ii. She will be paid for the number of bags of leaves she rakes as follows: two cents for filling one bag, four cents for filling two bags, eight cents for filling three bags, and so on, with the amount doubling for each additional bag filled.

Exp

- a. Is plan i. an example of linear or exponential growth? How do you know?

Ln because you add \$2

- b. Is plan ii. an example of linear or exponential growth? How do you know?

Exp because you multiply by 2

- c. If Celia rakes enough to five bags of leaves, should she opt for payment method 1 or 2? What if she fills ten bags of leaves?

i. $5 \cdot 2 = \$10$

ii. $.01(2)^5 = .32$

$10 \cdot 2 = \$20$

$.01(2)^{10} = 10.24$

Plan i is better.

- d. How many bags of leaves would Celia have to fill before method 2 pays more than method 1?

i. $y = 2x$
 ii. $y = .01(2)^x$

On the 12th bag plan ii is better (40.96 vs 24)

2. The population of a country is initially 2 million people and is increasing at 4% per year. The country's annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.

- a. Is the population growing linearly or exponentially? What about the food supply? How do you know?

Population is exponential because growing by a percent (Adds more each year)
 Food is linear because adds same amount each year.

- b. Based on these assumptions, in approximately what year will this country first experience shortages of food?

$$P(x) = 2(1.04)^x$$

$$F(x) = 4 + 0.5x$$

By year 23 the population will exceed the amount of food!
 (5.33 vs 5.25)

- c. If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year?

$$P(x) = 2(1.04)^x$$

$$F(x) = 8 + 0.5x$$

By year 42,
 $P(x) > F(x)$
 (10.386 vs 10.1)

- d. If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur?

$$P(x) = 2(1.04)^x$$

$$F(x) = 8 + x$$

By year 103
 $P(x) > F(x)$
 (113.62 vs 111)

3. If a pattern is changing by a percent rate, r , is it a linear or exponential pattern? How do you calculate the m ?

percent rate \rightarrow exponential
 $m = 1 + r$ (make sure to convert % to a decimal)