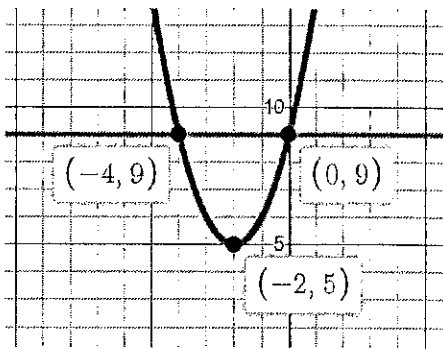
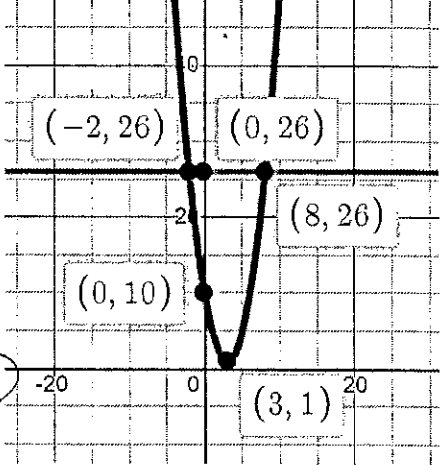
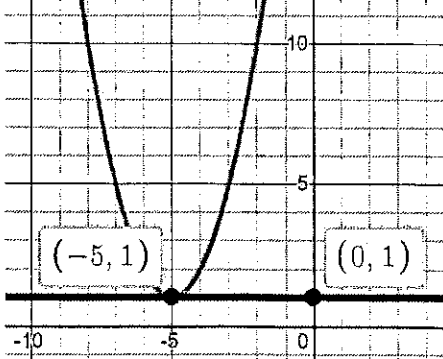


Completing the Square and the Quadratic Formula

This activity will lead you to the **QUADRATIC FORMULA**, which is one of the greatest achievements in the history of mathematics (seriously). The Quadratic Formula can be used to solve any quadratic equation. It is a very powerful tool.

Completing the Square Warm-up: Use CTS to rewrite each equation in VERTEX FORM. Then, solve for x (make sure to get ALL solutions). Use the graphs to check.

<p>1. $x^2 + 4x + 9 = 9$</p> $(x+2)^2 + 5 = 9$ $(x+2)^2 = 4$ $x+2 = 2 \quad x+2 = -2$ $x = 0 \quad x = -4$	
<p>2. $x^2 - 6x + 10 = 26$</p> $(x-3)^2 + 1 = 26$ $(x-3)^2 = 25$ $x-3 = 5 \quad x-3 = -5$ $x = 8 \quad x = -2$	
<p>3. $x^2 + 10x + 26 = 1$</p> $(x+5)^2 + 1 = 1$ $(x+5)^2 = 0$ $x+5 = 0$ $x = -5$	

4. A trick for CTS: Multiply by "4a". This "4a" trick makes quadratic equations with $a > 1$ easier to write in vertex form. Read the following example, then try the problems.

$2x^2 + 10x - 28 = 0$	$16x^2 + 80x - 224 = 0$	$(4x + 10)^2 - 324 = 0$
Multiply by $4a = 8$	Complete the Square	Vertex Form

Then you can solve for x

$(4x + 10)^2 = 324$	$4x + 10 = 18$ or -18	$4x = 8$ or -28	$x = 2$ or -7
Add 324	Square root 324	Subtract 10	Divide 4

5. Use the "4a" trick to solve for x. Some of the steps are done for you. $6x + 6$
 $\begin{array}{r} 6x \\ + \\ 6 \end{array} \begin{array}{r} 36x^2 \\ + \\ 36x \end{array} - 144$

$-3x^2 - 6x + 9 = 0$	$36x^2 + 72x - 108 = 0$	$(6x + 6)^2 - 144 = 0$
Multiply by $4a = -12$	Complete the Square	Vertex Form

Now you can solve for x because you isolated the variable.

$(6x + 6) = 144$	$6x + 6 = 12$ or -12	$6x = 6$ $6x = -18$	$x = 1, x = -3$
Add 144	Square root	Subtract 6	Divide 6

6. Use the "4a" trick to solve for x

$-16x^2 + 48x = 0$	$1024x^2 - 3072x = 0$	$(32x - 48)^2 - 2304 = 0$
Multiply by $4a = -64$	Complete the Square	Vertex Form

Now you can solve for x because you isolated the variable.

$(32x - 48)^2 = 2304$	$32x - 48 = -48$	$32x = 0$	$x = 0$ or $x = 3$
Add 2304	Sqrt 2304	Add 48	Divide 32

7. Completing the Square leads to the Quadratic Formula.

Do the problems side by side. At the end you will have a formula for the x-intercepts.

$-3x^2 - 9x + 12 = 0$

Multiply by $4a = -12$
 $36x^2 + 108x - 144 = 0$

Complete the Square

$6x + 9$	
$6x$	9
$36x^2$	$54x$
9	81

$-225 = 0$

Vertex Form

$(6x + 9)^2 - 225 = 0$
 $+225 +225$

Solve for x

$\sqrt{(6x + 9)^2} = \sqrt{225}$

$6x + 9 = 15$	$6x + 9 = -15$
$-9 -9$	$-9 -9$
$6x = 6$	$6x = -24$
<u>6</u>	<u>6</u>
$x = 1$	$x = -4$

$ax^2 + bx + c = 0$

Multiply by $4a =$
 $4a^2x^2 + 4abx + 4ac = 0$

Complete the Square

$2ax + b$	
$2ax$	b
$4a^2x^2$	$2abx$
$+ b$	b^2

$-b^2 + 4ac = 0$

Vertex Form

$(2ax + b)^2 - b^2 + 4ac = 0$
 $+b^2 -4ac +b^2 -4ac$

Solve for x

$\sqrt{(2ax + b)^2} = \sqrt{b^2 - 4ac}$

$2ax + b = \sqrt{b^2 - 4ac}$
 $-b -b$

$\frac{2ax}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

QUADRATIC FORMULA: IF $ax^2 + bx + c = 0$, THEN $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

8. Practice with the quadratic formula. Check that you found the x-intercepts by graphing.

• $2x^2 + 6x - 8 = 0$

$$X = \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(-8)}}{2(2)} = \frac{-6 \pm \sqrt{36 + 64}}{4}$$

$$= \frac{-6 \pm \sqrt{100}}{4} = \frac{-6 \pm 10}{4} = \frac{-16}{4} \text{ or } \frac{4}{4}$$

• $x^2 + 5x - 6 = 0$

$$X = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-6)}}{2(1)} = \frac{-5 \pm \sqrt{25 + 24}}{2} = \frac{-5 \pm \sqrt{49}}{2}$$

$$= \frac{-5 \pm 7}{2} = \frac{-12}{2} \text{ or } \frac{2}{2}$$

$X = -4 \text{ or } 1$

• $x^2 - 1 = 0$

$$X = \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(-1)}}{2(1)} = \frac{\pm \sqrt{4}}{2} = \pm \frac{2}{2} = 1 \text{ or } -1$$

~~$X = -6 \text{ or } 1$~~

• $x^2 + 8x = 0$

$$X = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(0)}}{2(1)} = \frac{-8 \pm \sqrt{64}}{2} = \frac{-8 \pm 8}{2} = 0 \text{ or } -8$$

• $-3x^2 - 4x - 5 = 0$

$$X = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-3)(-5)}}{2(-3)} = \frac{4 \pm \sqrt{16 - 60}}{-6}$$

No solution