

Compound Interest: Savings

While many financial investments are unpredictable and volatile, banks offer savings accounts that are incredibly secure and stable. As interest rates in the U.S. begin to rise, many people will return to savings accounts as a holding place for assets.

1. The table below shows the amount of money in a bank account, with \$100 initial balance, earning 3% (=0.03) annual interest.
Complete the table:

Years investment has been in the bank.	Balance at the beginning of the year	Interest earned during the year	Balance at the end of the year.
1	\$100	$\$100 \cdot 0.03 = \3	$\$100 + 3 = \103
2	\$103	$\$103 \cdot 0.03 = 3.09$	$\$103 + 3.09 = \106.09
3	\$106.09	$\$106.09 \cdot 0.03 = 3.18$	$\$106.09 + 3.18 = 109.27$
4	109.27	3.28	112.55
5	112.55	3.38	115.93

2. The table below shows the amount of money in a bank account, with \$500 initial balance, earning 4.5% (=0.045) annual interest.
Complete the table:

Years investment has been in the bank.	Balance at the beginning of the year	Interest earned during the year	Balance at the end of the year.
1	\$500	$\$500 \cdot 0.045 = \22.5	$\$500 + 22.5 = \522.50
2	\$522.50	$\$522.50 \cdot 0.045 = 23.51$	$\$522.50 + 23.51 = 546.01$
3	546.01	24.57	570.58
4	570.58	25.68	596.26
5	596.26	26.83	623.09

3. The table below shows the amount of money in a bank account with \$P initial balance earning r% (in decimal form) annual interest.
Complete the table:

Years investment has been in the bank.	Balance at the beginning of the year	Interest earned during the year	Balance at the end of the year.	Simplified end-of-year balance
1	\$P	\$Pr	$\$(P + Pr)$	$\$P(1+r)$
2	$\$P(1+r)$	$\$P(1+r) \cdot r$	$P(1+r) + P(1+r) \cdot r$	$\$P(1+r) \cdot (1+r) = P(1+r)^2$
3	$P(1+r)^2$	$P(1+r)^2 \cdot r$	$P(1+r)^2 + P(1+r)^2 \cdot r$	$P(1+r)^2(1+r)$
4	$P(1+r)^3$	$P(1+r)^3 \cdot r$	$P(1+r)^3 + P(1+r)^3 \cdot r$	$P(1+r)^3(1+r)$
5	$P(1+r)^4$	$P(1+r)^4 \cdot r$	$P(1+r)^4 + P(1+r)^4 \cdot r$	$P(1+r)^4(1+r)$

4. (a) Let t be the number of years the investment has been in the bank. Write an equation for the balance at the end of year balance, F, in terms of P, r and t.

$$F = P(1+r)^t$$

- (b) Use the equation to find the balance of a \$100 initial investment after 40 years in the bank with a 3% annual interest rate.

$$F = 100(1+0.03)^{40} = 326.20$$

Practice:

5. \$1000 is invested in an account earning 1.5% annual interest (meaning calculated once per year). Use the equation the value of this investment after 25 years.

$$F = 1000(1 + 0.015)^{25} = 1450.95$$

6. After 10 years, an initial investment of \$1000 had a balance of \$1283.03. Find the annual interest rate the account was earning.

$$\frac{1283.03}{1000} = \frac{1000}{1000} (1+r)^{10} \rightarrow 1.025 = 1+r$$

7. Consider two investments:

Investment A: Initial investment of \$10,000, annual interest rate of 7% invested for 40 years.

Investment B: Initial investment of \$100,000, annual interest rate of 7% invested for 5 years.

- (a) Predict which investment will have a greater ending balance. *B. Bigger beginning*

- (b) Use the equation for compound interest to determine the ending balance for each investment.

$$A = 10000(1 + 0.07)^{40} = 149,744.58$$

- (c) What does this result tell you about investment strategies?

$$B = 100000(1 + 0.07)^5 = 140255.17$$

Challenges:

Time is the most important factor in investments

8. Typically, banks calculate interest at the end of each month. Explain why the equation

$F = P(1 + \frac{r}{12})^{12t}$ represents this situation. Be specific. *Because they pay $\frac{1}{12}$ of the annual interest rate (r), & they pay 12 x per year*

9. Write an equation that banks would use if they compounded interest daily (365 days in a year).

$$F = P(1 + \frac{r}{365})^{365t}$$

10. Use what you learned from Desmos: What Comes Next? to find the rule that represents each table below:

x	0	5	10	15
y	2	4	8	16

~~$y = 2(2)^x$~~ $y = 2(2)^{x/5}$

x	0	3	6	9
y	0.1	1	10	100

$$y = 0.1(10)^{x/3}$$

x	0	10	20	30
y	7	19	31	43

$$y = \frac{12}{10}x + 7$$

x	0	2	4	6
y	48	12	3	0.75

$$y = 48(\frac{1}{4})^{x/2}$$