CCSS Advanced Algebra 3 Solving One-Variable Inequalities

A. Linear Inequalities

a. Consider the inequality $5x \le 30$. I know that $x = 2$ is a solution, because $5(2) = 10 < 30$. I know that $x = 10$ is NOT a solution, because $5(10) = 50 > 30$. Find 2 other solutions.

b. Consider the inequality $10x - 2 \le 18$. Find 3 solutions to this inequality. In other words, find 3 values of x that make the inequality true.

c. What is the largest value of x that solves the inequality? How do you know it is the largest possible solution? Is there a smallest possible solution? Why or why not?

d. Now consider the inequality $12 - 3x \le 15$. Find 3 solutions to this inequality.

e. What is the smallest value of x that solves the inequality above. How do you know it is the smallest possible solution? Is there a largest possible solution? Why or why not?

f. Now consider the inequality $-3 \le 4 - x \le 2$. Find 3 solutions to this inequality.

g. What is the smallest value of x that solves the inequality above. How do you know it is the smallest possible solution? Is there a largest possible solution? Why or why not?

h. A **boundary point** is the smallest (or largest depending on whether it is a < or > problem) solution to an inequality. A **test point** can be used to check to see where solutions lie relative to the boundary point. How can you use the boundary and test points to write all solutions to an inequality? See Solving Linear Inequalities for further guidance.

i. Practice: Solve each inequality below. Write your solution using inequality symbols ($<, \ >, \ \le, \ \ge$) and graphically on a number line.

- $5 - 2x \ge 1$
- $4x - 3 < 10x + 21$
- $2(x-1) - 3 > -11$
- $5 - (x - 3) \le 18$
- $4 \le 10 - 2x \le 18$

B. *Quadratic Inequalities*

a. Consider the inequalities below:

i. $x^2 < 9$ ii. $(x+2)^2 > 16$ iii. $2(x-4)^2 + 3 \le 53$

How could you find the boundary points for each inequality? How many boundary points will these inequalities have? How many test points do you need to use in this case?

b. Solve for the **boundary points** by treating each inequality like it was an equation.

i. $x^2 = 9$ ii. $(x+2)^2 = 16$ iii. $2(x-4)^2 + 3 = 53$

c. Solve each inequality by using **test points** to determine the solution intervals. (see example below if you get stuck)

Example: Solve $2(x+3)^2 - 4 > 4$

Step 1: Boundary point: Solve $2(x+3)^2 - 4 = 4$

$2(x+3)^2 = 8$

$(x+3)^2 = 4$

$x + 3 = 2$ or $x + 3 = -2$, so $x = -1$ or $x = -5$

Step 2: Test points: Choose a value in each possible solution interval.

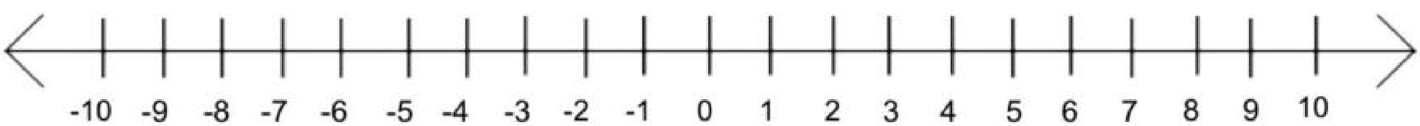
Use x = -10, x = -3 and x = 0

$2(-10+3)^2 - 4 = 94 > 4$ $2(-3+3)^2 - 4 = -4 < 4$ $2(0+3)^2 - 4 = 14 > 4$

So values in the first and third intervals make the inequality true. The solutions are

$x < -5,\ and\ x > -1$

Shade the appropriate region on the number line



Practice:

1. $(x-4)^2 + 8 < 17$

2. $3(x+5)^2 - 1 \le 11$

C. Other functions

a. Consider the inequalities below:

i. $2\sqrt{x-3} + 1 > 5$ ii. $-4|x-3| - 2 \le -10$ iii. $\frac{4}{x} - 2 = \frac{5}{2x}$

How could you find the boundary points for each inequality? How many boundary points will each type of inequality have?

b. For problem a.i., why does it make sense that the solution should be in the form $3 \le x < \#$? Explain your answer fully.

c. Extra practice

- $2(x-1)^3 - 1 \ge 3$

- $|x+3| - 1 < 0$

- $-2\sqrt[3]{x} + 7 > 9$

- $\frac{2}{x+1} - 1 \le \frac{5}{x-1}$

- $\frac{2}{x+1} - x \leq \frac{4}{x-1}$