CCSS Advanced Algebra 4 The Special Role of Zero in Factoring

In your table groups, discuss and solve Questions 1 and 2 below:

1. Find all real solutions to the equation:
   a. $(x^2 + 5x + 6)(x^2 - 3x - 4) = 0$.
   b. $(x^2 - 9)(x^2 - 16) = 0$.

2. Suppose we know that the polynomial equation $4x^3 - 12x^2 + 3x + 5 = 0$ has three real solutions and that one of the factors of $4x^3 - 12x^2 + 3x + 5$ is $(x - 1)$. How can we find all three solutions to the given equation?

Complete Exercises 3–7 in your math notebooks:

3. Find the real zeros (also known as roots or x-intercepts) of the following polynomial functions.
   a. $f(x) = (x + 1)(x - 1)(x^2 + 1)$

   b. $g(x) = (x - 4)^3(x - 2)^8$

   c. $h(x) = (2x - 3)^5$

   d. $k(x) = (3x + 4)^{100}(x - 17)^4$

4. A Zero or Root of a Polynomial can have a **multiplicity** if the root is repeated in the function. For example, $m(x) = (x - 2)^4(x + 1)^2$ has two roots: $x = 2$ with **multiplicity** 4 and $x = -1$ with **multiplicity** 2. Find the multiplicity for each root in Question 3.

5. Write a polynomial function that has the following zeros and multiplicities. What is the degree of your polynomial?

Zero	Multiplicity
2	3
-4	1
6	6
-8	10

6. Is there more than one polynomial function that has the same zeros and multiplicities as the one you found in Exercise 5?

7. Can you find a rule that relates the multiplicities of the zeros to the degree of the polynomial function?

Problem Set

For Problems 1–4, find all solutions to the given equations.

1. $(x-3)(x+2)=0$

2. $(x-5)(x+2)(x+3)=0$

3. $(2x-4)(x+5)=0$

4. $(2x-2)(3x+1)(x-1)=0$

5. Find four solutions to the equation $(x^2-1)(x^2-36)=0$.

6. Find two different polynomial functions that have zeros at 1, 3, and 5 of multiplicity 1.

7. Find two different polynomial functions that have a zero at 2 of multiplicity 5 and a zero at -4 of multiplicity 3.

8. If p, q, r, s are nonzero numbers, find the solutions to the equation $(px+q)(rx+s)=0$ in terms of p, q, r, s.

Use the identity $a^2-b^2=(a-b)(a+b)$ to solve the equations given in Problems 9-10.

9. $(x-3)(x+3)=(2x-1)(2x+1)$

10. $(3x+2)(3x-2)=(2+3x)(2-3x)$