We have been exploring the idea of compound interest recently. Banks calculate interest payments to customers at regular intervals (typically monthly) but in many situations, growth occurs continuously, not at regular distinct intervals. In this activity, we will explore the idea of perpetual exponential growth or decay.

1. Different Compounding Frequencies
   a. Consider an earlier situation in which Emily is investing $1000 in an account earning 3% annual interest. How much would Emily have in her account after 1 year?

   b. Now assume that Emily's bank is calculating interest every 6 months (twice a year). Explain why Emily would have $1000(1+\frac{0.03}{2})^2 = \$1030.225$ in her account after 1 year.

   c. What if Emily's bank was calculating interest every month (which is typical). How much would she have in her account after one year? Show how you found your answer.

   d. Why does it make sense that her end-of-the-year balance is greater the more frequently the bank calculates interest?

2. The Thought Experiment:
   In 1683, the mathematician Jacob Bernoulli considered the following question:

An account starts with $1.00 and pays 100% interest per year. If the interest is credited once, at the end of the year, the value of the account at year-end will be $2.00. What happens if the interest is computed and credited more frequently during the year?

   a. Use what you learned from Question 1 to determine the value of Bernoulli's fictional account after one year if interest in computed:
      i. Every month. ii. Every day.

      iii. Every hour. iv. Every minute.

      v. Every second.

   b. What do you notice about the values you calculated in part (a)?

c. Mathematicians were so fascinated by the result that they defined the symbol e (known as Euler's number and readily found on calculators) as the answer to Bernoulli's thought experiment: if an infinite number of interest calculations were performed in a year. In other words, if interest was calculated perpetually.

i. Why does it make sense that your answer to a.v. is an approximation to e?

ii. What would you do to find a more accurate approximation to e?

3. Why Should We Care?

a. Human Population Growth

One of the problems we've looked at this week is how populations of cities or countries have grown over time. For example, the U.S. Population (in millions) was represented by the function $P(x) = 226 \cdot 1.00957^x$, where $x =$ years after 1980.

i. Use this equation (and our new found understanding of logarithms) to find the number of years before the 1980 U.S. population doubled. Show how you found your answer.

ii. This model assumes that population growth occurs annually. What might be wrong with this assumption?

iii. Does it make sense that a large population, like the U.S., would experience perpetual growth? Why or why not?

iv. A perpetual growth model for the U.S. would look like $Q(x) = 226 \cdot e^{0.00957x}$, where $x =$ years after 1980. Use this model (and logarithms) to find the number of years before the 1980 population doubles.

v. Why does it make sense that your answer to iv is smaller than i?

b. Radioactive Material

A little more than 200 miles from Portland is the Hanford Nuclear Site. This facility, when operational, produced millions of gallons of plutonium waste that is currently being stored and processed. Plutonium decays very, very slowly.

i. Why does it make sense that radioactive decay is best modeled a perpetual decay model?

ii. The decay of 10 pounds of plutonium would be modeled by the function $d(x) = 10 \cdot e^{-0.00002888x}$, where $x =$ # of years of decay. Use the function to determine the number of years it takes for half of the plutonium to decay (this is called the half-life of plutonium).