

5. Rational $y = a\left(\frac{1}{x-h}\right) + k$

- a. A rational function's asymptotes intersect at the point (3, 5). The curve passes through the point (4, 4).

$$y = a\left(\frac{1}{x-3}\right) + 5 \rightarrow \begin{matrix} y = a\left(\frac{1}{4-3}\right) + 5 \\ y = a(1) + 5 \\ -1 = a \end{matrix} \quad y = -1\left(\frac{1}{x-3}\right) + 5$$

- b. The relationship between pressure and volume of a gas in a rigid container, held at constant temperature, can be modeled with a rational function. If the volume of a container of air is 100 ml, then the pressure is .78 kJ/cm². If the volume is reduced to 50 ml, then the pressure increases to 1.56 kJ/cm². (Think about what the asymptotes MUST be!)

$$y = a\left(\frac{1}{x}\right) \quad \begin{matrix} .78 = a\left(\frac{1}{100}\right) \\ 78 = a \end{matrix} \quad 1.56 = 78\left(\frac{1}{50}\right) \text{ checks out.}$$

6. Exponential $y = a(b)^x$

- a. An exponential function has an asymptote of $y = 1$ and passes through the points

(0, 5) and (3, 17)
 (0, 4) (3, 16) $y = 4(4)^{x-3} + 1$

Times 4.

- b. Mr. Maurer is growing bacteria for some insidious science experiment. There are 24 bacteria in a culture after 3 hours, and after 6 hours there are 192 bacteria.

$$\begin{matrix} 24 = a(b)^3 \\ 192 = a(b)^6 \end{matrix} \rightarrow \begin{matrix} 8 = b^3 \\ 2 = b \end{matrix} \quad \begin{matrix} 24 = a(8) \\ 3 = a \end{matrix} \quad y = 3(2)^x$$

7. Sine/Cosine

$$y = a \sin(b(x+c)) + d \quad \text{OR} \quad y = a \cos(b(x+c)) + d$$

- a. A trigonometric function has a maximum of (1, 5) and a minimum of (3, 1). Find as many functions as apply.

$$y = 2 \sin(90x) + 3 \quad \text{OR} \quad y = 2 \cos(90(x-90)) + 3$$

- b. A car tire is approximately 24 inches in diameter. The car is traveling at such a speed that the tire makes 3 rotations every second.

$$y = 12 \sin(120x) + 12$$

8. Sine/Cosine

- a. A trigonometric function has a midline of $y = -3$, an amplitude of 2, and a period of 5. Find as many functions as apply

$$y = 2 \sin\left(\frac{360}{5}x\right) - 3$$

- b. On its shortest day, December 20th, Portland gets about 6 hours of sunlight. On its longest day, June 20th, Portland gets about 18 hours. Find as many functions as apply.

$$y = 6 \cos\left(\frac{360}{365}(x-20)\right) + 12$$