Discontinuities and End Behavior Practice

For each rational function, list the following: roots, holes, asymptotes, and end behavior (both positive and negative). Then sketch a graph.

Reminders: **Roots** are the factors of the numerators that don't cancel out. **Holes** are the factors of both the numerator and denominator that cancel. **Asymptotes** are the factors of the denominator that did not cancel. **End behavior** is what the y-values of your function approach as x gets really really really big or really really really small (positive/negative infinity).

$y = \frac{x}{x^2-x-6} = \frac{x}{(x-3)(x+2)}$

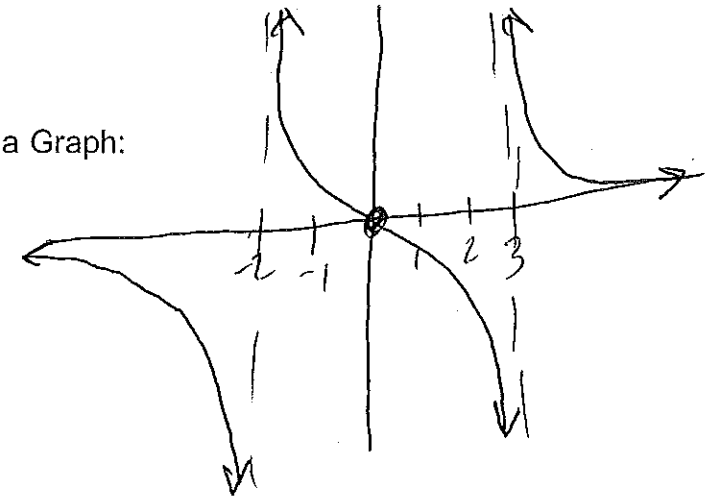
Roots: $x=0$

Holes: None

Asymptotes: $x=3, x=-2$

End Behavior: $\lim_{x\to\infty} y = \lim_{x\to\infty} y = 0$

Sketch a Graph:



neg | pos | neg | pos

$(-)(-)$ -2 $(-)(+)$ 0 $(-)(+)$ 3 $(+)(+)$

$y = \frac{x^2+8x-20}{x^2-x-6} = \frac{(x+10)(x-2)}{(x-3)(x+2)}$

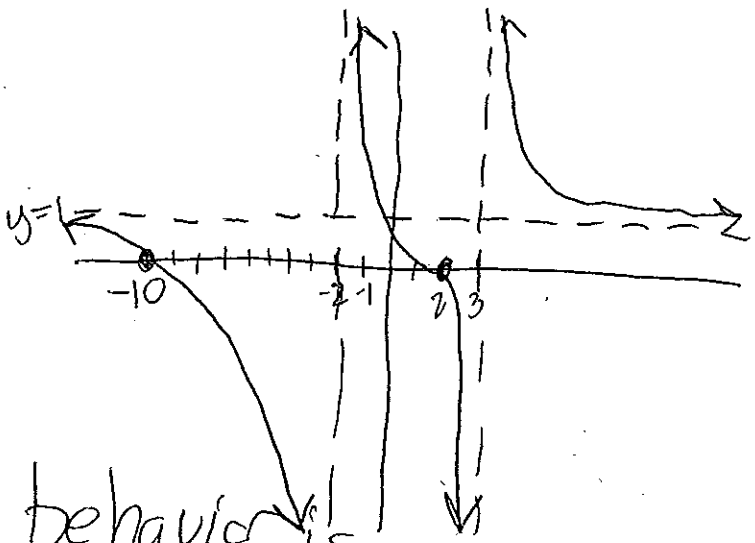
Roots: $x=-10, x=2$

Holes: None

Asymptotes: $x=3, x=-2$

End Behavior: $\lim_{x\to\infty} y = \lim_{x\to\infty} y = 1$

Sketch a Graph:



pos | neg | pos | neg | pos

$(-)(-)$ -10 $(+)(-)$ -2 $(-)(+)$ 2 $(+)(+)$ 3 $(+)(+)$

Note: The end behavior is like an asymptote except you can cross it for small x.

QF $x = \frac{2 \pm \sqrt{4-4(1)(-4)}}{2} = \frac{2 \pm \sqrt{4+16}}{2} = \frac{2 \pm \sqrt{20}}{2}$

$y = \frac{2x^2-4x-8}{3x^2-27} = \frac{2(x^2-2x-4)}{3(x^2-9)} = \frac{2(x-(1+\sqrt{5}))(x-(1-\sqrt{5}))}{3(x+3)(x-3)}$ $= \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$

Roots: $1 \pm \sqrt{5}$

Holes: None

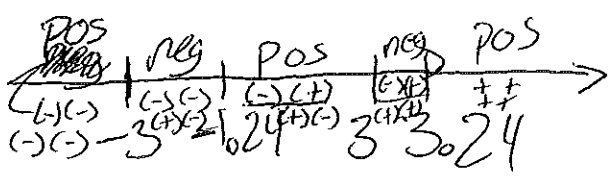
Asymptotes: $x=3, x=-3$

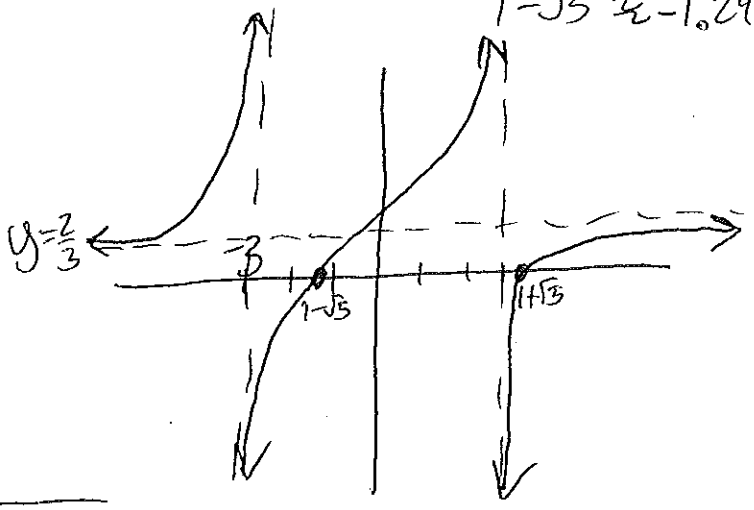
End Behavior: $\lim_{x \to \infty} y = \lim_{x \to -\infty} y = \frac{2}{3}$

Sketch a Graph:

$1+\sqrt{5} \approx 3.24$

$1-\sqrt{5} \approx -1.24$





$y = \frac{x^2}{4x^2-100} = \frac{x^2}{4(x^2-25)} = \frac{x^2}{4(x+5)(x-5)}$

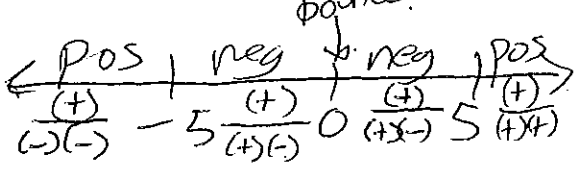
Roots: $x=0$ ← Should bounce.

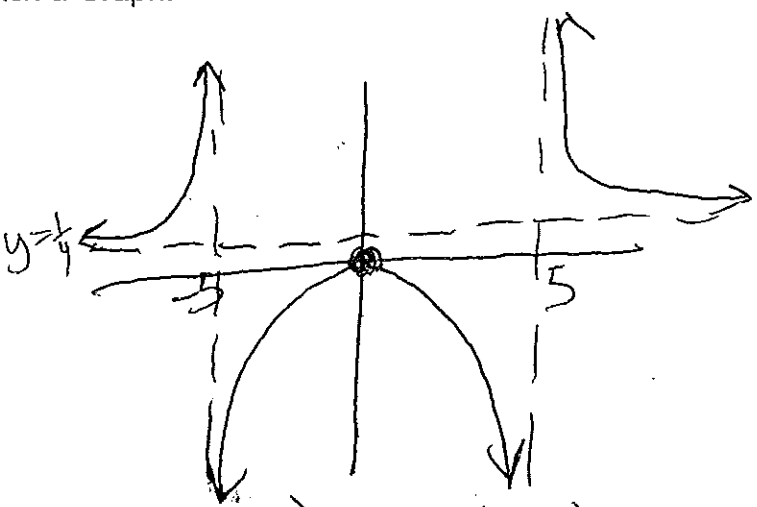
Holes: None

Asymptotes: 5 & -5

End Behavior: $\lim_{x \to \infty} y = \lim_{x \to -\infty} y = \frac{1}{4}$

Sketch a Graph:





$y = \frac{64-x^2}{x^2-5x-24} = \frac{(8-x)(8+x)}{(x-8)(x+3)} = \frac{-1(x+8)(x+8)}{(x-8)(x+3)} = \frac{-1(x+8)}{(x+3)}$

Roots: $x=-8$

Holes: $x=8$

Asymptotes: $x=-3$

End Behavior: $\lim_{x \to \infty} y = \lim_{x \to -\infty} y = -1$

Sketch a Graph: