End Behavior

End Behavior measures the y-values of a function as x moves toward positive/negative infinity (The far left/right ends of the function)

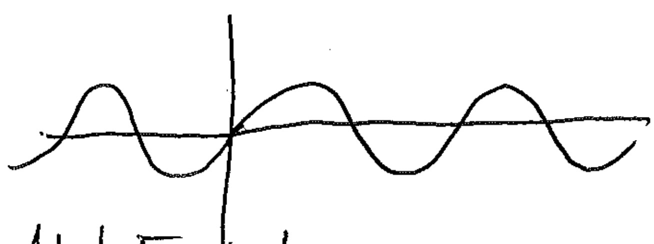
If you have a graph, just move far to the right & left and focus on the y-values.

The y-values will either approach $\pm\infty$, approach a distinct y-value, or oscillate continuously between a range of values.

The End Behavior is written as

$$\lim_{x \to \pm\infty} f(x) = L$$

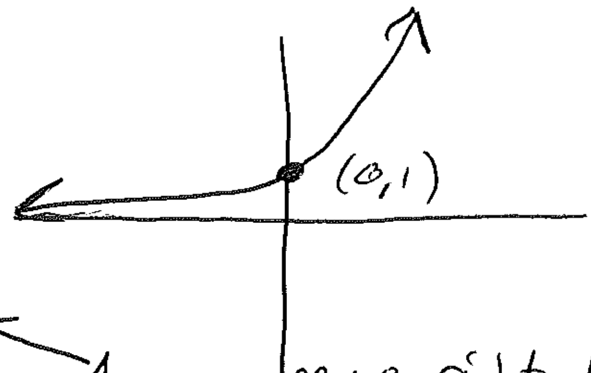
Examples:

1) $f(x) = \sin x$ 

$\lim_{x \to \pm\infty} \sin x$ Does Not Exist.

The values never balance out. They always oscillate between 1 & -1, no matter how far you move left or right. Because there's no single value, the limit does not exist (like ManGirls)

2) $g(x) = 3^x$



$\lim_{x \to \infty} 3^x = \infty$

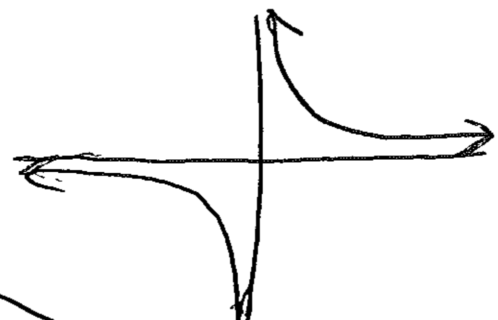
As you move right the y-values get really big.

$\lim_{x \to -\infty} 3^x = 0$

As you move left, the y-values get really close to zero.

Recall that $3^{-2} = \frac{1}{3^2}$ so negative powers approach $y=0$ (the x-axis)

3) $h(x) = \frac{1}{x}$

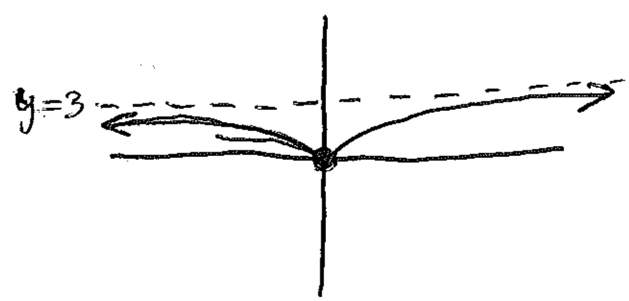


$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to -\infty} \frac{1}{x} = 0$

I can move right or left, but my y-values approach zero in either direction.

4) $r(x) = \frac{3x^2}{x^2+1}$



$\lim_{x \to \pm\infty} r(x) = 3$

We are studying rational functions, which have easily predictable end behavior. The important feature is the degrees of the numerator & denominator

Remember: Degree = Highest power of x.

There are 3 situations:

Top > Bottom	Top < Bottom	Top = Bottom
$\lim_{x \to \pm\infty} f(x) = \pm\infty$	$\lim_{x \to \pm\infty} f(x) = 0$	$\lim_{x \to \pm\infty} f(x) = \frac{a}{b}$
Fractions with big numerators are big numbers	Fractions with big denominators are close to zero	Fractions with same size top & bottom "balance out" to a ratio

1) $f(x) = \dfrac{3x^3 + 2x^2 - 7}{2x^2 + 15x - 913}$

Top > Bottom. $\lim_{x \to \infty} f(x) = \infty$

x^3 is neg, x^2 is pos, so negative. $\longrightarrow$ $\lim_{x \to -\infty} f(x) = -\infty$

2) $g(x) = \dfrac{17x^3 + 15x^2 - 143x}{\frac{1}{10}x^4}$

Bottom > Top. $\lim_{x \to \pm\infty} = 0$

3) $h(x) = \dfrac{4x^2 + 3x - 4}{2x^2 + 1}$

Bottom = Top.

Ratio of leading coefficients.

$\lim_{x \to \pm\infty} h(x) = \frac{4}{2} = 2$