CCSS Advanced Algebra 4 Exponential and Logarithmic Applications

1. A Super Ball rebounds to $\frac{3}{4}$ of its previous height after each bounce. If you drop a Super Ball from a height of 20 m, after how many bounces will it reach a height of 2 m? Solve using an exponential model and logarithms.

$y = 20(3/4)^x$

$\frac{2}{20} = \frac{20(3/4)^x}{20}$

$\frac{1}{10} = (3/4)^x$

$\log_{(3/4)}(1/10) = x = \frac{\log(1/10)}{\log(3/4)} = 8.004$

Between 8 & 9.

2. Reproduction of an African Dung Beetle is the focus of a laboratory experiment. There were 25 Dung Beetles at the beginning of the experiment. It was noted that the number of Dung Beetles increase 3% every 28 days. After how many days will there be 500 Dung Beetles? Solve using an exponential model and logarithms.

$y = 25(1.03)^{x/28}$

$\frac{500}{25} = \frac{25(1.03)^{x/28}}{25}$

$20 = (1.03)^{x/28}$

$\log_{1.03} 20 = x/28$

$28 \cdot \log_{1.03}(20) = 28 \cdot 101.35 = 2837.75$

3. Two rival companies: Acme Lighting and Bargain Bulbs decided to make the same LED light bulbs using two different processes. The revenue (in thousands of $) of the two companies are represented by: $a(t) = 1000\,log_4 t + 100$ and $b(t) = 1200\,log_5 t$ where t = time in months.

i. How many months will it take for Acme Lighting to have $5,000,000 in revenue? Show how you found your answer.

$5000000 = 1000\log_4 t + 100$
$-100 \qquad -100$

$4{,}999{,}900 = 1000\log_4 t$

$4{,}999.9 = \log_4 t$

$4^{4.9999} = t = 1023.86$

ii. How many months will it take Bargain Bulbs to have $5,000,000 in revenue? Show how you found your answer.

$\frac{5000}{1200} = \frac{1200\log_5 t}{1200}$

$4.16 = \log_5 t$

$5^{4.16} = t = 817.29$

iii. Using the graphs of $a(t)$ and $b(t)$, after how many months will Acme Lighting and Bargain Bulbs have the same revenue?

$x = 61.742$, $y = 3074.09$, 61.742 months

iv. Find the inverse of $a(t)$.

$a^{-1}(t) = 4^{\frac{t-100}{1000}}$

v. Evaluate $a^{-1}(5,000,000)$. What does this mean about Acme Lighting's revenue?

$a^{-1}(5{,}000{,}000) = 1023.86$ (See problem 3i)

After 1023 months, the revenue is 5,000,000

4. Radium (Ra) is a radioactive element that decays as follows: In 3,000 years, a 100 gram sample of radium decays to a mass of 27.04 grams.

a. Write an exponential function to describe the decay of radium over time. Define your variables.

$y = 100\left(\frac{27.04}{100}\right)^{x/3000}$ y = grams of Ra, x = years OR $y = 100(.2704)^{x/3000}$

b. Find the inverse of the exponential function from part (a).

$y^{-1} = \log_{.2704}(x/100) \cdot 3000$

c. Use the inverse function to determine the number of years it would take a sample of radium to decay to half of its original mass.

$\log_{.2704}\left(\frac{50}{100}\right) \cdot 3000 = 1589.97$ years

5. The towns of Geometrix and Matrix are matched for a cultural exchange. The population of Geometrix is 40,000 while the population of Matrix is 10,000. For the next 30 years, experts predict that the population of Geometrix will decline by 3% per year. During the same period, they expect that the population of Matrix will increase by 5% annually.

$x = 17.49$ years

a. Find after how many years the two towns will have the same population graphically.

$G(x) = 40{,}000(.97)^x,\ M(x) = 10{,}000(1.05)^x$

b. How many years ago did Geometrix have a population of 10,000? Show how you found your answer.

$\frac{40{,}000(.97)^x}{40000} = \frac{10{,}000}{40000}$

$(.97)^x = 1/4$

$x = \log_{.97}(1/4)$

$x = 45.5$ years

6. When interest is paid n times a year, the value of an initial investment, P, that collects an annual interest rate of r (*as a decimal*) for x years can be represented by the function

$C(x) = P(1 + \frac{r}{n})^{nx}$

Ella wants to invest \$2000. She has two investment options:

Investment Option A $C(x) = 2000(1.05)^x$	Investment Option B $C(x) = 2000(1 + \frac{.042}{12})^{12x}$
• Annual Interest Rate of 5% • Interest Paid Once per Year	• Annual Interest Rate of 4.2% • Interest Paid Monthly (12 times per year)

Using the graphs of the functions representing each Investment Option, determine after how many months, the two Investments Options will have the same balance?

$x = 0$. Other than at the start, A is better than B.

7. An airplane is flying at an altitude of 10,000 meters. At 21:00, the pilot begins the descent towards PDX Airport. The descent follows an exponential model, $d(x)$, ending with the plane's landing. At 21:04, the airplane is an an altitude of 5,222 meters.

$21{:}00 \Rightarrow x = 0,\quad 21{:}04 \rightarrow x = 4$

$\frac{5222}{10{,}000} = 0.5222$

a. At what time will the plane be at 280 meters? Solve using logarithms.

$d(x) = 10000(.5222)^{x/4}$

$\frac{280}{10000} = \frac{10000(.5222)^{x/4}}{10000}$

$.028 = (.5222)^{x/4}$

$\log_{.5222} .028 = x/4 \rightarrow 5.5 = x/4$ $x = 22$

b. Find the inverse of the exponential function that represents the airplane's descent, $d^{-1}(x)$.

$d^{-1}(x) = 4 \cdot \log_{.5222}(x/10000)$

c. Evaluate $d^{-1}(280)$. What does this tell you about the airplane's descent?

$d^{-1}(280) \approx 22$ (see 7a)

After 22 minutes, the plane is 280 meters above the ground.