Sequences and Exponential Review Packet

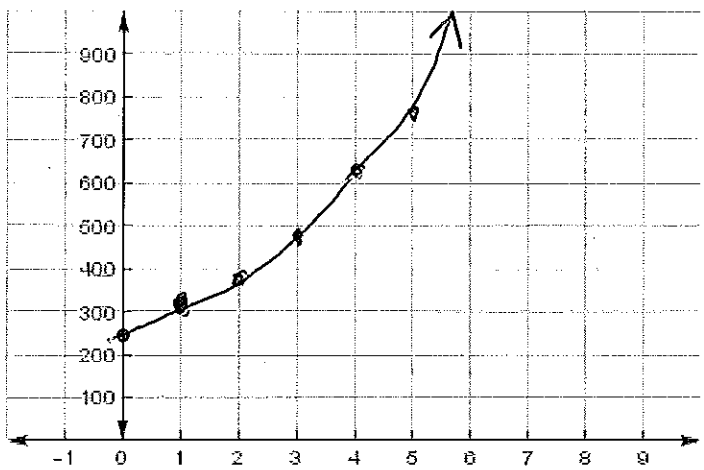
Name: KEY

1. a. You are trying to grow a large enough army of bacteria to take over the world. You start with 250 bacteria, and the population increases by 25% every day. Write an equation to model this situation. Define your variables.

$y = 250(1.25)^x$

y = bacteria, x = days

b. Sketch a graph of the situation.



c. Create a table for the situation

x	y
0	250
1	312.5
2	390.625
3	488.28
4	610.35
5	762.94

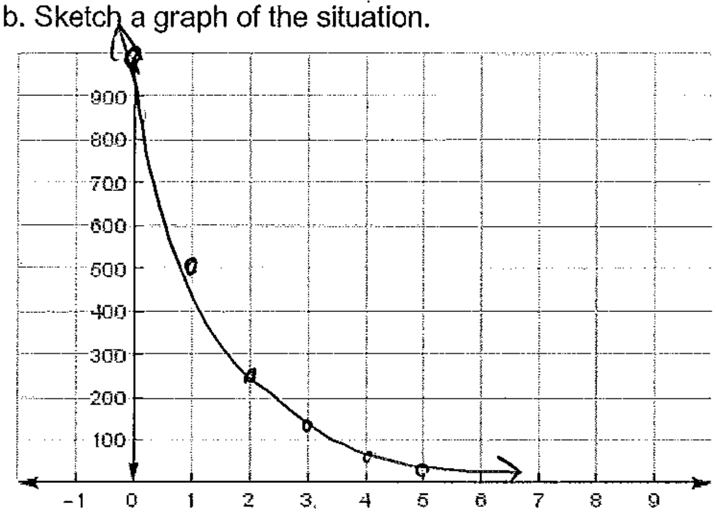
d. How long until you have your army of 10,000,000 bacteria?

47.5 ≈ 48 days

2. a. You are trying to determine the age of an ancient artifact. You know that carbon-14 has a half-life of around 1570 years. Your artifact started with 1000 mg of carbon-14. Let x = # of half-lifes.

$y = 1000(.5)^x$

b. Sketch a graph of the situation.



c. Create a table for the situation

x	y
0	1000
1	500
2	250
3	125
4	62.5
5	31.25

d.If your artifact now has 62.5 mg of carbon-14, how old is it?

4 halflifes, or $4 \cdot 1570 = 6280$ years

3. a. You are looking at two savings plans and trying to choose the best plan for saving for college. The first plan pays 7% annual interest, compounded monthly. The second plan pays 7.2% annual interest, compounded annually. Make a table for each plan if you start with $500. Include at least 5 entries in your table.

1st Plan: $y = 500\left(1 + \frac{.07}{12}\right)^{12x}$

2nd Plan: $y = 500(1 + .072)^{x}$

x = years
y = money

1st Plan

x	y
0	500
1	536.145
2	574.90
3	616.46
4	661.03

2nd plan

x	y
0	500
1	536
2	574.59
3	615.96
4	660.31

b. Write an exponential model for each plan.

c. Which plan would you choose? Why?

The first. It is higher at each time.

4. a. You are looking at two savings plans and trying to choose the best plan for saving for college. The first plan pays 5% annual interest, compounded quarterly. The second plan pays 7.2% annual interest, compounded daily. Make a table for each plan if you start with $500. Include at least 5 entries in your table.

x = years
y = money

1st Plan

x	y
0	500
1	525.47
2	552.24
3	580.38
4	609.94

2nd Plan

x	y
0	500
1	537.32
2	577.43
3	620.54
4	666.86

b. Write an exponential model for each plan.

1st: $y = 500\left(1 + \frac{.05}{4}\right)^{4x}$

2nd: $y = 500\left(1 + \frac{.072}{365}\right)^{365x}$

c. Which plan would you choose? Why?

The 2nd plan, it is more $$.

5. Simplify the following expressions so they only contain positive exponents.

a. $(4x^3)(-3x^6)(2x^{-2})$

$-4\cdot 3\cdot 2x^{3+6+-2} = \boxed{-24x^7}$

b. $\frac{15x^4y^5z^3}{5x^2y^{-3}z^3} = 3x^{4-2}y^{5-(-3)}z^{3-3}$

$\boxed{3x^2y^8}$

c. $\left(\frac{36x^3y^7}{4x^{-3}y^4}\right)^2 = \left(9x^{3-(-3)}y^{7-4}\right)^2$

$(9x^6y^3)^2 = \boxed{81x^{12}y^6}$

d. $\left(\frac{64x^3y^6z^7}{16x^{-4}y^6z^5}\right)^{-2} = \left(4x^{3-(-4)}y^{6-6}z^{7-5}\right)^{-2}$

$(4x^7z^2)^{-2} = \boxed{\frac{1}{16x^{14}z^4}}$

6. Explain why the following equations make sense, using the definition of an exponent:

Repeated multiplication

a. $x^3x^6x^8 = x^{17}$

$x^3\cdot x^6\cdot x^8 = \underbrace{(x\cdot x\cdot x)}_{3}\underbrace{(x\cdot x\cdot x\cdot x\cdot x\cdot x)}_{6}\underbrace{(x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x)}_{8}$

$= x^{17}$ because $3+6+8=17$.

b. $\frac{x^7}{x^3} = x^4$

$\frac{x^7}{x^3} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x\cdot x}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}} = x^4$

c. $(2x^3)^3 = 8x^9$

$(2x^3)^3 = (2x^3)(2x^3)(2x^3) = 2\cdot 2\cdot 2\cdot x^3\cdot x^3\cdot x^3$

$= 8\cdot\underbrace{(x\cdot x\cdot x)(x\cdot x\cdot x)(x\cdot x\cdot x)}_{x^9} = 8\cdot x^9$

d. $\frac{1}{x^6} = x^{-6}$

Dividing is the opposite of multiplying, positives are opposites of negatives. Because positive exponents mean multiply, negatives mean the opposite, divide.

7. Let $f(x) = \dfrac{2+3x}{x-1}$, and $g(x) = \dfrac{2+x}{x-3}$

a. Find f(g(x))

$$\frac{2+3g(x)}{g(x)-1} = \frac{2+3\left(\frac{2+x}{x-3}\right)}{\frac{2+x}{x-3}-1}\left(\frac{x-3}{x-3}\right)$$

b. Find g(f(x))

$$\frac{2(x-3)+3(2+x)}{2+x \quad -1(x-3)} = \frac{2x-6+6+3x}{2+x \quad -x+3} \rightarrow \frac{5x}{5} = x.$$

$$\frac{2+f(x)}{f(x)-3} = \frac{2+\frac{2+3x}{x-1}}{\frac{2+3x}{x-1}-3}\left(\frac{x-1}{x-1}\right) = \frac{2(x-1)+2+3x}{2+3x-3(x-1)} = \frac{2x-2+2+3x}{2+3x-3x+3}$$

$$= \frac{5x}{5} = x.$$

c. Are f and g inverses?

Yes!

8. Let $f(x) = \dfrac{3+2x}{x+1}$, and $g(x) = \dfrac{x+1}{3+2x}$

a. Find f(g(x)) $\Rightarrow \dfrac{3+2g(x)}{g(x)+1} = \dfrac{3+2\left(\frac{x+1}{3+2x}\right)}{\frac{x+1}{3+2x}+1}\left(\dfrac{3+2x}{3+2x}\right) = \dfrac{3(3+2x)+2(x+1)}{x+1+1(3+2x)}$

$$= \frac{9+6x+2x+2}{x+1+3+2x} = \boxed{\frac{11+8x}{4+3x}}$$

b. Find g(f(x))

$$\frac{f(x)+1}{3+2f(x)} = \frac{\frac{3+2x}{x+1}+1}{3+2\left(\frac{3+2x}{x+1}\right)}\left(\frac{x+1}{x+1}\right) = \frac{3+2x+x+1}{\underset{3x+3}{3(x+1)}+\underset{6+4x}{2(3+2x)}} = \boxed{\frac{4+3x}{9+7x}}$$

c. Are f and g inverses?

No!

9. Let f(x) = 3 + 4x, and g(x) = 3x - 4

a. Find f(g(x))

$$3+4g(x) = 3+4(3x-4) = 3+12x-16$$
$$\boxed{= 12x-13}$$

b. Find g(f(x))

$$3f(x)-4 = 3(3+4x)-4 = 9+12x-4 = \boxed{12x-5}$$

c. Are f and g inverses?

No!

8. Complete the table to match the first 4 terms of the sequence with its explicit and recursive formulae.

Sequence	Explicit	Recursive
-4, -1, 2, 5...	$f(x) = -4 + 3(x-1)$	$f(x+1) = f(x) + 3$; $f(1) = -4$
27, 9, 3, 1	$f(x) = 27\,(1/3)^{x-1}$	$f(x+1) = f(x)(1/3)$; $f(1) = 27$
3, 6, 12, 24	$f(x) = 3(2)^{x-1}$	$f(x+1) = 2f(x)$; $f(1) = 3$
0, 2, 8, 18	$f(x) = 2(x-1)^2$	$f(x+1) = f(x) + 4x - 2$; $f(1) = 0$
2, 4, 8, 16...	$f(x) = 2 \cdot 2^{x-1}$ OR 2^x	$f(x+1) = f(x) \cdot 2$; $f(1) = 2$
-5, -2, 1, 4	$f(x) = -5 + 3(x-1)$	$f(x+1) = f(x) + 3$; $f(1) = -5$
-2, 1, 4, 7	$f(x) = -2 + 3(x-1)$	$f(x+1) = f(x) + 3$; $f(1) = -2$
24, 8, 8/3...	$f(x) = 24(1/3)^{x-1}$	$f(x+1) = f(x) \cdot \frac{1}{3}$; $f(1) = 24$

9. Mr. Maurer finds two terms from a sequence, and all he knows is that the sequence is geometric. The two terms are 1 and 729. Continue the sequence in each situation.

a. 1, 729, 531,441, 387,420,489

b. 1, 27, 729, 19683

c. 1, 9, 81, 729

d. 1, 3, 9, 27, 81, 243, 729

e. 531441, 729, 1, 1/729

10. A piece of toilet paper is .15 mm thick. The milky way is approximately 9.46 x 10^{20} m across.

$.15\text{mm} = .15 \div 1000\text{ m} = .00015\text{m}$

a. How many times do you need to fold your toilet paper in half to reach across the milky way?

y = thickness, x = # of folds

$y = \frac{.15}{1000}(2)^{x-1} = .00015(2)^{x-1}$

83 times

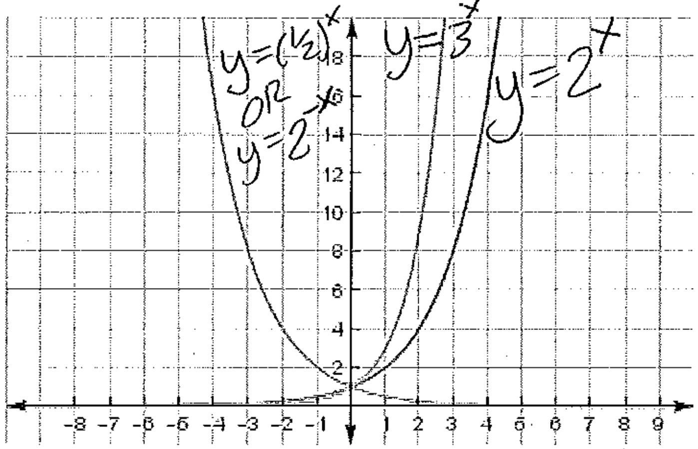
x	y
0	.00015
1	.0003
2	.0006
3	.0012
4	.0024
5	.0048
etc.	
82	7.25×10^{20}
83	1.45×10^{21}

11. What does it mean for an exponent to be a fraction? Use an example in your explanation. Integer exponents tell you to multiply by the base. Fraction exponents only multiply a fraction of the way, which is like multiplying by a root of the base

Ex $9^0 = 1$, $9^1 = 9$, $9^2 = 9\cdot9 = 81$, $9^3 = 9\cdot9\cdot9 = 723$

$\sqrt{9} = 3$, $9^{1/2} = 3$, $9^{1\frac{1}{2}} = 3\cdot9 = 27$, $9^{2\frac{1}{2}} = 3\cdot81 = 243$

12. Write the equation for each graph



$y = (\frac{1}{2})^x$

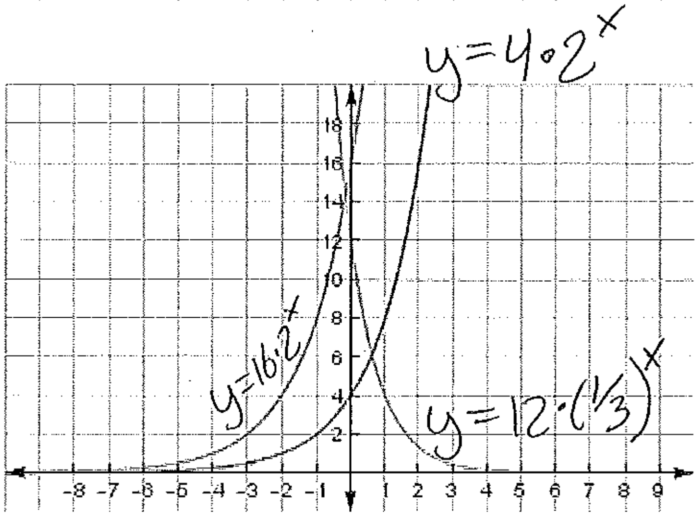
x	y
-2	4
-1	2
0	1
1	1/2
2	1/4

$y = 3^x$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

$y = 2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4



$y = 16\cdot2^x$

x	y
-2	4
-1	8
0	16
1	32
2	64

$y = 4\cdot2^x$

x	y
-2	1
-1	2
0	4
1	8
2	16

$y = 12(\frac{1}{3})^x$

x	y
-2	108
-1	36
0	12
1	4
2	4/3