

# Factoring

Factoring is perhaps the most important skill you learn in Algebra. It's definitely the most important skill for polynomials.

Factoring is reversing the distributive property

$$3(x+5) = 3 \cdot x + 3 \cdot 5 = 3x + 15$$

$$3 \begin{array}{|c|c|} \hline x & 5 \\ \hline 3x & 15 \\ \hline \end{array}$$

← Notice that the factors are on the outside; the terms on the inside.

Reminder: "Factors" are things that are multiplied  
"Terms" are things that are added

Distributing is when you multiply out the factors and combine like terms.

Factoring is when you split the terms and write your expression as a product of factors.

Factoring is most useful for finding the zeros of polynomials.

Example:  $x^2 + 6x + 8 = (x+2)(x+4)$

It is hard to solve  $x^2 + 6x + 8 = 0$

$$\begin{array}{|c|c|} \hline x & 2 \\ \hline 4 & 8 \\ \hline \end{array}$$

It is easy to solve  $(x+2)(x+4) = 0$   
 $x = -2$  or  $x = -4$   
because  $0 \cdot x = 0$ , no matter what "x" is.

How can you deal with both " $x^2$ " & " $6x$ "?

When the polynomials get more complicated, use the Integral Roots Theorem and polynomial division to factor.

Ex |  $x^3 - 15x^2 + 39x + 55$

IRT: Possible roots are  $\pm 1, \pm 5, \pm 11, \pm 55$

$x = -1$  is a root because  $(-1)^3 - 15(-1)^2 + 39(-1) + 55 = 0$   
 So  $x + 1$  is a factor. Use polynomial division

$x$	$x^3$	$-16x^2$	$55x$	→	$x^2 - 16x + 55$ $(x-11)(x-5)$
$+1$	$x^2$	$-16x$	$55$		
$-1$	$x^3 - 15x^2 + 39x + 55$				

So it can be fully factored as  $(x+1)(x-11)(x-5)$

Ex2 | You can also factor out multiple roots at a time.

$$x^4 + 4x^3 - 37x^2 - 64x + 336$$

IRT: Possible roots are  $\pm 1, 2, 3, 4, 6, 7, 16, 21, \text{etc.}$

$x = 4$  &  $x = -4$  are both roots (you can check for yourself)

So,  $(x-4)$  &  $(x+4)$  are factors. Thus  $(x-4)(x+4) = x^2 - 16$ .

$x^2$	$x^4$	$4x^3$	$-21x^2$	→	$(x^2 - 16)(x^2 + 4x - 21)$ $(x+4)(x-4)(x+7)(x-3)$
$-16$	$-16x^2$	$-64x$	$336$		
	$x^4 + 4x^3 - 37x^2 - 64x + 336$				