

HW: Function Notation

Function notation is a useful way in mathematics to identify different equations. We use it as a formal way to show whether to EVALUATE (find the value) of a function, or SOLVE (for a variable).

Part 1: Use the following functions to answer the problems below:

$$f(x) = 2x - 3$$

$$g(x) = \frac{-12}{x}$$

$$m(x) = x^2$$

$$d(x) = 2(x - 3)$$

Here we will practice EVALUATING.

Example 1: Find $f(-3)$.

$$f(-3) = 2(-3) - 3$$

$$f(-3) = -6 - 3$$

$$f(-3) = -9$$

Example 2: Find $d(6)$.

$$d(6) = 2(6 - 3)$$

$$d(6) = 2(3)$$

$$d(6) = 6$$

You Try:

1. Find $f(5)$.

$$\begin{aligned} f(5) &= 2(5) - 3 \\ &= 10 - 3 \\ f(5) &= 7 \end{aligned}$$

4. Find $m(5)$.

$$\begin{aligned} m(5) &= (5)^2 \\ m(5) &= 25 \end{aligned}$$

7. Find $f(-4)$.

$$\begin{aligned} f(-4) &= 2(-4) - 3 \\ f(-4) &= -11 \end{aligned}$$

2. Find $g(-2)$.

$$\begin{aligned} g(-2) &= \frac{-12}{-2} \\ g(-2) &= 6 \end{aligned}$$

5. Find $m(-5)$.

$$\begin{aligned} m(-5) &= (-5)^2 \\ m(-5) &= 25 \end{aligned}$$

8. Find $d(11)$.

$$\begin{aligned} d(11) &= 2(11 - 3) \\ d(11) &= 16 \end{aligned}$$

3. Find $d(-5)$.

$$\begin{aligned} d(-5) &= 2(-5 - 3) \\ &= 2(-8) \\ d(-5) &= -16 \end{aligned}$$

6. Find $g(6)$.

$$\begin{aligned} g(6) &= \frac{-12}{6} \\ g(6) &= -2 \end{aligned}$$

9. Find $m(-11)$.

$$\begin{aligned} m(-11) &= (-11)^2 \\ m(-11) &= 121 \end{aligned}$$

Part 2: Now, we will practice SOLVING using the following functions to answer the problems below:

$$f(x) = 2x - 3$$

$$g(x) = \frac{-12}{x}$$

$$d(x) = 2(x - 3)$$

Example 1: Solve $f(x) = -11$

$$2x - 3 = -11$$

$$2x = -8$$

$$x = -4$$

Example 2: Solve $g(x) = 6$.

$$\frac{-12}{x} = 6$$

$$-12 = 6x$$

$$-2 = x$$

You Try:

1. Solve $d(x) = -18$

$$\begin{aligned} 2(x - 3) &= -18 \\ \frac{2(x - 3)}{2} &= \frac{-18}{2} \\ x - 3 &= -9 \\ x &= -6 \end{aligned}$$

2. Solve $f(x) = 15$.

$$\begin{aligned} 2x - 3 &= 15 \\ +3 +3 & \\ 2x &= 18 \\ x &= 9 \end{aligned}$$

3. Solve $g(x) = 3$.

$$\begin{aligned} \frac{-12}{x} &= 3 \\ -12 &= 3x \\ -4 &= x \end{aligned}$$

4. Solve $d(x) = -22$.

$$\begin{aligned} 2(x - 3) &= -22 \\ x - 3 &= -11 \\ x &= -8 \end{aligned}$$

Part 3: More Challenging Mixed Practice:

5. If $m(x) = x^2$, solve $m(x) = 144$

$x^2 = 144$
 $x = 12$ $x = -12$

6. If $h(x) = x^2 - 5x + 3$, find $h(-7)$.

$(-7)^2 - 5(-7) + 3$
 $49 + 35 + 3$
 $84 + 3 = 87$

7. If $p(x) = \frac{2x-5}{3}$, find $p(18)$.

$\frac{2(18)-5}{3} = \frac{36-5}{3} = \frac{31}{3}$

8. If $p(x) = \frac{2x-5}{3}$, solve $p(x) = -5$.

$\frac{2x-5}{3} = -5$
 $2x-5 = -15$
 $2x = -10 \rightarrow x = -5$

Part 4: Finding & Checking Inverses:

$f(x) = 2x - 3$

$g(x) = \frac{-3x+2}{5}$

$h(x) = -3 + 2(x+1)^3$

$k(x) = 3\sqrt{x+4} - 2$

To find an inverse, you set the function equal to "y" and solve for "x" using SADMEP. Swap the "x" and "y" of the final result to write the inverse as a function. To check if two functions are inverses, look at a table of values to see that the domain and range are switched¹.

Example: Find the inverse of $f(x)$, i.e. find $f^{-1}(x)$

| | | | | | | | | | | | | | | | | |
|--|--|----|------|-----|-----|----|------|----|----|---|-----|---------------------|-----|---|------|----|
| $y = 2x - 3$ $y + 3 = 2x$ $\frac{y+3}{2} = x$ $f^{-1}(x) = \frac{x+3}{2}$ | <table border="1"> <tr> <td>x</td> <td>0</td> <td>-3</td> <td>1.5</td> <td>-9</td> </tr> <tr> <td>f(x)</td> <td>-3</td> <td>-9</td> <td>0</td> <td>-21</td> </tr> <tr> <td>f⁻¹(x)</td> <td>1.5</td> <td>0</td> <td>2.25</td> <td>-3</td> </tr> </table> | x | 0 | -3 | 1.5 | -9 | f(x) | -3 | -9 | 0 | -21 | f ⁻¹ (x) | 1.5 | 0 | 2.25 | -3 |
| x | 0 | -3 | 1.5 | -9 | | | | | | | | | | | | |
| f(x) | -3 | -9 | 0 | -21 | | | | | | | | | | | | |
| f ⁻¹ (x) | 1.5 | 0 | 2.25 | -3 | | | | | | | | | | | | |

Explain how the table supports the fact that the two functions are inverses

Because if $f(a) = b$ then $f^{-1}(b) = a$.

Examples: $f(0) = -3 \rightarrow f^{-1}(-3) = 0$
 $f(-3) = -9 \rightarrow f^{-1}(-9) = -3$

Practice:

1. Find $g^{-1}(x)$ and check with a table

$-3x + 2 = y$
 $-3x + 2 = 5y$
 $-3x = 5y - 2$

$x = \frac{5y-2}{-3}$
 $g^{-1}(x) = \frac{5x-2}{-3}$

| | | |
|----|------|---------------------|
| x | g(x) | g ⁻¹ (x) |
| 4 | -2 | -6 |
| 6 | 4 | -32/3 |
| -2 | 9/5 | 4 |

2. Find $h^{-1}(x)$ and check with a table

$y = -3 + 2(x+1)^3$
 $y + 3 = 2(x+1)^3$
 $\frac{y+3}{2} = (x+1)^3$
 $\sqrt[3]{\frac{y+3}{2}} = x+1$

| | | |
|-----|-------|---------------------|
| x | h(x) | h ⁻¹ (x) |
| 0 | -1 | .145 |
| -1 | -3 | 0 |
| -3 | -19 | -1 |
| -19 | -1651 | -3 |

3. Find $k^{-1}(x)$ and check with a table

$y = 3\sqrt{x+4} - 2$
 $y + 2 = 3\sqrt{x+4}$
 $\frac{y+2}{3} = \sqrt{x+4}$

$(\frac{y+2}{3})^2 = x+4$
 $(\frac{y+2}{3})^2 - 4 = x$
 $k^{-1}(x) = (\frac{x+2}{3})^2 - 4$

| | | |
|---|------|---------------------|
| x | k(x) | k ⁻¹ (x) |
| 0 | 4 | -5.5 |
| 4 | 6.49 | 0 |

$\sqrt[3]{\frac{y+3}{2}} - 1 = x$
 $h^{-1}(x) = \sqrt[3]{\frac{x+3}{2}} - 1$

¹ Later this unit we will use **composition of functions** to check more formally if two functions are inverses. If this worksheet is going easily for you, please ask Maurer about **function composition**.