CCSS Advanced Algebra 4 AA8_6: Introduction to the Normal Distribution

1. **Heights of Dinosaurs and the Normal Curve**

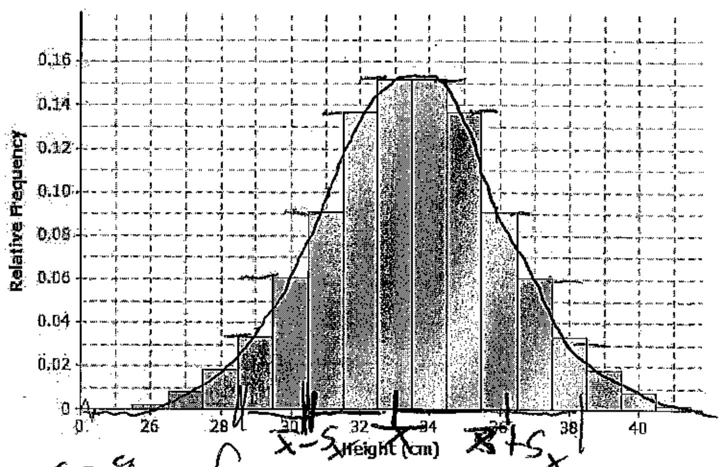
A paleontologist studies prehistoric life and sometimes works with dinosaur fossils. The table below shows the distribution of heights (rounded to the nearest centimeter) of 660 procompsognathids, otherwise known as COMPYS. The heights were determined by studying the fossil remains of the compys.

Height (cm)	Number of Compys	Relative Frequency
26	1	0.002
27	5	0.008
28	12	0.018
29	22	0.033
30	40	0.061
31	60	0.091
32	90	0.136
33	100	0.152
34	100	0.152
35	90	0.136
36	60	0.091
37	40	0.061
38	22	0.033
39	12	0.018
40	5	0.008
41	1	0.002
Total	660	1.000

The following is a RELATIVE FREQUENCY HISTOGRAMS of the compy heights:

a. What does the relative frequency of 0.136 mean for the height of $32\ cm$?

13.6% of compys are 32cm tall



b. What is the width of each bar? What does the height of the bar represent?

width = 1

height = relative frequency

d. The mean of the distribution of compy heights is $33.5\ cm$, and the standard deviation is $2.56\ cm$. Interpret the mean and standard deviation in this context. Average height is 33.5. 68% of compys are within 2.56cm of average

e. Mark the mean on the graph, and mark one deviation above and below the mean.

i. Approximately what percent of the values in this data set are within one standard deviation of the mean (i.e., between $33.5\ cm - 2.56\ cm = 30.94\ cm$ and $33.5\ cm + 2.56\ cm = 36.06\ cm$)?

About 68% in theory

9 + 13 + 15 + 15 + 13 + 9

73

ii. Approximately what percent of the values in this data set are within two standard deviations of the mean?

In theory, 95% For this data 73+6+6+3+3=91

f. Draw a smooth curve that comes reasonably close to passing through the midpoints of the tops of the bars in the histogram. Describe the shape of the distribution.

Symmetrical, normal

g. Shade the area of the histogram that represents the proportion of heights that are within one standard deviation of the mean.

h. Based on our analysis, how would you answer the question, "How tall was a compy?"

Between 31 & 36 cm tall

2. **Gas Mileage and the Normal Distribution**

A NORMAL CURVE is a smooth curve that is symmetric and bell shaped. Data distributions that are mound shaped are often modeled using a NORMAL CURVE, and we say that such a distribution is approximately NORMAL. One example of a distribution that is approximately normal is the distribution of compy heights from Example 1. Distributions that are approximately normal occur in many different settings.

For example, a salesman kept track of the gas mileage for his car over a 25-week span. The mileages (miles per gallon rounded to the nearest whole number) were
23, 27, 27, 28, 25, 26, 25, 29, 26, 27, 24, 26, 26, 24, 27, 25, 28, 25, 26, 25, 29, 26, 27, 24, 26.

a. Consider the following:

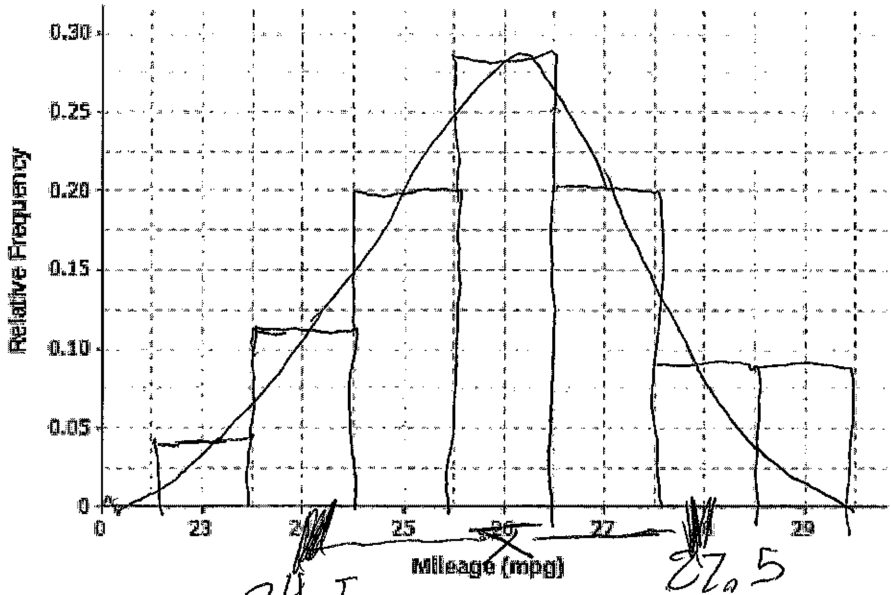
i. Use technology to find the mean and standard deviation of the mileage data.

$\bar{X} = 26.04$, $S_x = 1.54$

ii. Calculate the relative frequency of each of the mileage values. For example, the mileage of 26 *mpg* has a frequency of 7. To find the relative frequency, divide 7 by 25, the total number of mileages recorded. Complete the following table:

Mileage (mpg)	Frequency	Relative Frequency
23	1	.04
24	3	.12
25	5	.20
26	7	.28
27	5	.20
28	2	.08
29	2	.08
Total	25	1

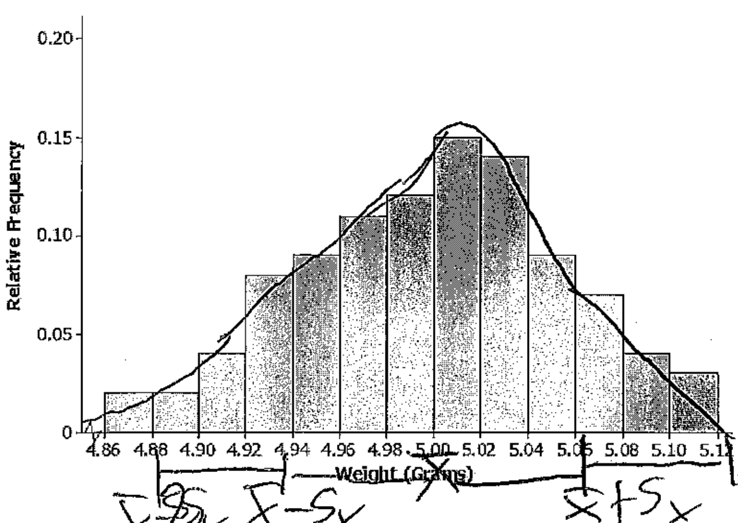
iii. Construct a relative frequency histogram using the scale below.



a. Describe the shape of the mileage distribution. Draw a smooth curve that comes reasonably close to passing through the MIDPOINTS of the tops of the bars in the histogram. Is this approximately a normal curve? Yes. Symmetrical

b. Mark the mean on the histogram. Mark one standard deviation to the left and right of the mean. Shade the area of the histogram that represents the proportion of mileages that are within one standard deviation of the mean. Find the proportion of the data within one standard deviation of the mean. 68% in theory $\frac{12}{2} + 20 + 28 + 20 + \frac{8}{2} = 78$

3. Periodically the U.S. Mint checks the weight of newly minted nickels. Below is a histogram of the weights (in grams) of a random sample of 100 new nickels.

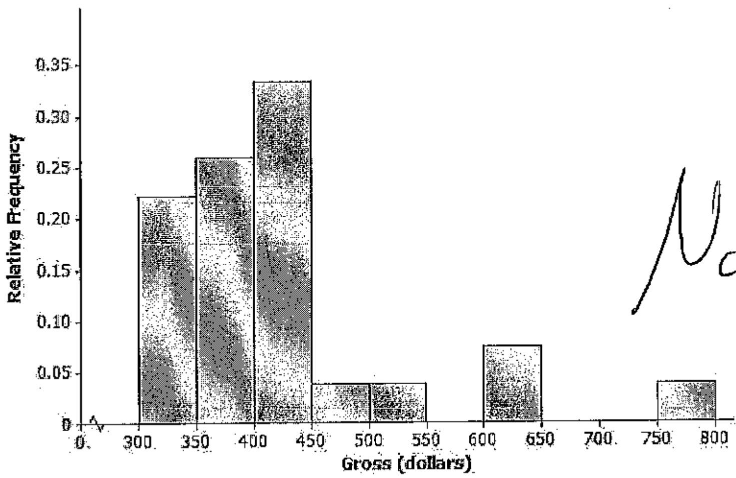


a. The mean and standard deviation of the distribution of nickel weights are 5.00 grams and 0.06 gram, respectively. Mark the mean on the histogram. Mark two standard deviations above the mean and two standard deviations below the mean.

b. Describe the shape of the distribution. Draw a smooth curve that comes reasonably close to passing through the midpoints of the tops of the bars in the histogram. Is this approximately a normal curve? Approximately, but with a left skew

c. Shade the area of the histogram that represents the proportion of weights that are within Two standard deviations of the mean. Find the proportion of the data within two standard deviations of the mean. Looks like 98%. Theory says 95%

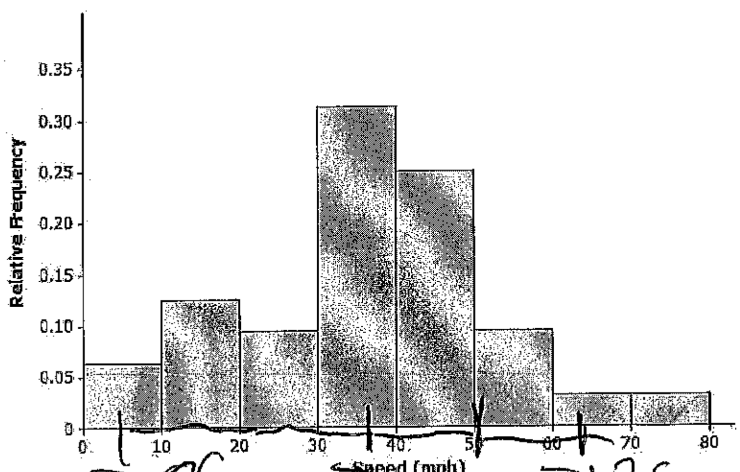
4. Below is a relative frequency histogram of the gross (in millions of dollars) for the all-time top-grossing American movies (as of the end of 2012). Gross is the total amount of money made before subtracting out expenses, like advertising costs and actors' salaries.



Not normal. Has a significant right skew

Describe the shape of the distribution of all-time top-grossing movies. Would a normal curve be the best curve to model this distribution? Explain your answer.

5. Below is a histogram of the top speed of different types of animals.



$\bar{x}-2S_x$ $\bar{x}$ $\bar{x}+2S_x$

Approximately normal

a. Describe the shape of the top speed distribution.

b. Draw a smooth curve that is approximately a normal curve. The actual mean and standard deviation of this data set are $34.1\ mph$ and $15.3\ mph$, respectively. Shade the area of the histogram that represents the proportion of speeds that are within two standard deviation of the mean. Show this is approximately 95% of the distribution.

Touches almost everything except last box