CCSS Advanced Algebra 4 Introduction to Complex Numbers

In the mid 1500s, an Italian mathematician named Gerolamo Cardano posed a simple sounding algebraic problem:

Find two numbers that add to 10 and multiply to 40.

This simple premise lead to the invention of a new set of numbers that have turned out to be interesting and useful in the development of deeper mathematical thought.

1. Let x and y be the numbers in Cardano's problem. Write two equations to represent the problem that he posed.

2. Solve this system of equations to find values of x and y. Is there a problem with the solutions you found? Explain why or why not.

3. Definitions:
   - Imaginary Unit: $i = \sqrt{-1}$
   - Complex Number: any number that can be expressed using the Imaginary Unit. For example, $5 + 2i$, $-3i$, $5 - \sqrt{15}i$.
   - Real Part of a Complex Number: the value of a complex number that does not include i.
   - Imaginary Part of a Complex Number: the value of a complex number that doe include i.

   a. Use the Quadratic Formula to solve the equation $x^2 + x + 1 = 0$. Determine the Real and Imaginary parts of each solution.

   b. Use the Quadratic Formula to solve the equation $6x^2 - 6x + 2 = 0$. Determine the Real and Imaginary parts of each solution.

   c. A quadratic function has complex roots $x = 2 - i$ and $x = 2 + i$. Write the function in Standard Form.

d. The polynomial function $p(x) = x^3 + 4x^2 + 11x + 8$ has a zero at $x = -1$. Find the two complex roots of $p(x)$.

4. Notes to Self:

- How can you use the Quadratic Formula to determine whether or not a quadratic function has complex roots?
- How can you use the Complex Roots of a parabola to write the function in Standard Form?
- If an n^{th} degree polynomial has 2 real roots, how many complex roots must it have? If it has 3 real roots, how many complex roots must it have?

5. Extra Practice:

a. Find all the roots (real and complex) of each polynomial:

i. $f(x) = x^2 + x + 11$ ii. $g(x) = (x^2 - 4)(x^2 + x + 2)$ iii. $h(x) = (x^2 - 2x - 8)(x^2 + 9)$

b. Solve each equation below (find real and complex solutions):

i. $-5x^2 = 3x + 10$ ii. $(x^2 + 3x + 5)(x - 4)^2(2x + 1) = 0$ iii. $(x^2 + 25)(x^2 - 1)(x^2 + 9) = 0$

6. Review (Rational Expressions): Simplify each expression below:

a. $\frac{3x}{x+1} + \frac{3}{x+1}$ b. $\frac{x^2}{x+3} - \frac{9}{x+3}$ c. $\frac{1}{x-1} + \frac{1}{x+1}$

d. $\frac{x}{x-3} \cdot \frac{(x-3)(x+3)}{x^2}$ e. $\frac{(x^2-25)(x^2+6x+5)(x-1)}{(x^2+10x+25)(x^2-1)}$