The inverse of a function undoes the function. All functions can be inverted (however it is not true that the inverse is always a function. Don't worry about that for now, we'll get there in a few weeks).

Always keep in mind the idea of "undoing." The intuitive way to find an inverse is to list the operations of the original function, then undo them one at a time to make the inverse.

My metaphor is getting ready to leave for work in the morning, and coming home from work at night.

Morning:

1. Put on socks,
2. Put on shoes,
3. Tie shoelaces,
4. Walk from home to car,
5. Drive car from home to CHS,
6. Walk into CHS

Night:

1. Walk out of CHS
2. Drive car from CHS to home
3. Walk from car to home
4. Untie shoelaces
5. Take off shoes
6. Take off socks

Please notice that the order of operations is exactly reversed, and that each nighttime operation undoes the corresponding daytime operation.

Here's a more mathy example:

f(x)= $\frac{(x-5)}{3}+11$

Operations:

1. Subtract 5
2. Divide 3
3. Add 11

Inverse:

1. Subtract 11
2. Multiply 3
3. Add 5

g(x) = (x - 11)*3 + 5 = 3(x - 11) + 5

You can verify that two functions are inverses by composing them with each other. (If you don't remember function composition, please go read that document before continuing.) The big idea of an inverse is that it undoes the function.

That means that if f(x) and g(x) are inverses, then f(g(x)) = x and g(f(x)) = x. (The "f" and "g" undo each other).

Let's verify our intuitive example:

$$f(x) = \frac{x-5}{3} + 11 \qquad g(x) = 3(x-11)+5$$

$$f(g(x)) = \frac{g(x)-5}{3} + 11 = \frac{3(x-11)+5-5}{3} + 11$$

$$= \frac{\cancel{3}(x-11)}{\cancel{3}} + 11 = x - 11 + 11 = x.$$

$$g(f(x)) = 3(f(x) - 11) + 5 = 3\left(\frac{x-5}{3} + 11 - 11\right) + 5$$

$$= \cancel{3}\left(\frac{x-5}{\cancel{3}}\right) + 5 = x - 5 + 5 = x.$$

Bonus Example :)

$$f(x) = \frac{2x+3}{5x-1} \qquad g(x) = \frac{x+3}{5x-2}$$

$$f(g(x)) = \frac{2g(x)+3}{5g(x)-1} = \frac{2\left(\frac{x+3}{5x-2}\right)+3}{5\left(\frac{x+3}{5x-2}\right)-1}\left(\frac{5x-2}{5x-2}\right)$$

$$= \frac{2\left(\frac{x+3}{\cancel{5x-2}}\right)\cancel{(5x-2)} + 3(5x-2)}{5\left(\frac{x+3}{\cancel{5x-2}}\right)\cancel{(5x-2)} - 1(5x-2)} = \frac{2(x+3)+3(5x-2)}{5(x+3)-1(5x-2)}$$

$$= \frac{2x+6+15x-6}{5x+15-5x+2} = \frac{\cancel{17}x}{\cancel{17}} = x.$$

$$g(f(x)) = \frac{f(x)+3}{5f(x)-2} = \left(\frac{\frac{2x+3}{5x-1}+3}{5\left(\frac{2x+3}{5x-1}\right)-2}\right)\left(\frac{5x-1}{5x-1}\right) = \frac{\left(\frac{2x+3}{\cancel{5x-1}}\right)\cancel{(5x-1)}+3(5x-1)}{5\left(\frac{2x+3}{\cancel{5x-1}}\right)\cancel{(5x-1)}-2(5x-1)}$$

$$= \frac{2x+3+15x-3}{5(2x+3)-2(5x-1)} = \frac{17x}{10x+15-10x+2} = \frac{\cancel{17}x}{\cancel{17}} = x.$$