CCSS Advanced Algebra 4 Language of Logarithms

1. Emily is hatching a plan to buy a house and move to Bora Bora. She knows (based on her newly acquired knowledge of compound interest) that if she invests now, her money will grow over time using the equation $V(x) = P(1+r)^x$, ;where $V(x)$ =value of her money after x years, P = initial amount invested and r = the interest rate she will earn.

   a. Explain why the calculation uses $1+r$ to calculate the growth in her account

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   b. Emily does research and finds that one of the highest yielding stocks for 2017 (Information Technology) paid 15% annual return. If she invested $1000 now at that rate, what would be the value of her investment after 20 years?

   c. Okay, so it turns out that Bora Bora real estate is expensive: http://www.sothebysrealty.com/eng/sales/pyf . Emily figures that she should be able to get something decent for $500,000. Determine how much she would need to invest now (still at 15% for 20 years) to afford a $500,000 property.

   d. That seems like an unreasonable amount. How about this: What interest rate would she need to earn to turn her $1000 into enough money to afford a $500,000 property after 20 years?

   e. Hmmm...I don't think there has ever been an investment that pays that percent of return. How many years would Emily need to keep her investment of $1000 at 15% to afford a $500,000 property? (Why can't you solve this equation like the previous equations?)

2. Mathematics is mostly a language. Symbols have been invented to mean particular things just like words and punctuation. Many of the symbols are so familiar to people that they don't have to think about them. Use what you know about the mathematical symbols used below to solve each equation:

a. $x+5=6$	b. $3x=12$	c. $\frac{x}{2}=10$
x=1	x=4	x= 20
d. $x^3=8$	e. $3^x=9$	f. $2^x=16$
x=2	x=2	x=4

3. Sometimes, the language of mathematics requires additional inventions (symbols) to solve equations. A good example of this is exponents/roots. Roots were defined as the inverse of raising different values to a given power, as follows:

$y = x^n$ is equivalent to $x = \sqrt[n]{y}$

Convert each equation below to the root version. Then use a calculator to solve for x:

a. $x^4 = 5.0625$	b. $x^{2.5} = 1024$	c. $x^{\frac{1}{4}} = 81$
x=1.5	x=16	x=43046721

Convert each equation below to the exponent version. Then use a calculator to solve for x:

d. $\sqrt[-2]{x} = 25$ e. $\sqrt[12]{x} = 1$ d. $\sqrt[\frac{1}{5}]{x} = 32$

x=0.0016 x=1 x=32

4. Referring back to question 1(e),

a. Explain why the equation you need to solve is

$$500000 = 1000(1.15)^x$$

b. The issue with solving this equation is that you haven't yet learned the language for reversing an equation when the exponent is a variable… until now!

Logarithms[1] are defined as the inverse of raising a given number to various exponents as follows:

$y = a^x$ is equivalent to $x = log_a y$, where a is called the base of the logarithm.

Convert each equation below to the logarithm version and use a calculator to solve for x.

i. $3^x = 243$ ii. $10^x = 0.01$ iii. $2.4^x = 33.1776$

x=5 x=

Convert each equation to the exponential version and determine the value of x.

iv. $log_3 x = 2$ v. $log_5 25 = x$ vi. $log_2 x = 0$

c. Return to question 1(e) and solve the equation using logarithms.

5. Are each of the following statements TRUE or FALSE? Explain your response:

a. $log_3 9 = 2$

b. $log_{10} x = 0$ has No Solution

c. $y = 4 \cdot 5^x$ is the inverse of $y = log_5(\frac{x}{4})$

d. These are the correct steps to solve:

$10 \cdot 5^{x-1} + 2 = 252$ (subtract 2)

$10 \cdot 5^{x-1} = 250$ (divide by 10)

$5^{x-1} = 25$ (convert to logarithms)

$log_5 25 = x - 1$ (add 1)

$3 = x$

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[1] Logarithms weren't invented until 1614 and, before the use of calculators, relied on tables constructed by meticulous calculations using base 10 logarithms. Today, scientific calculators will calculate logarithms of any base.