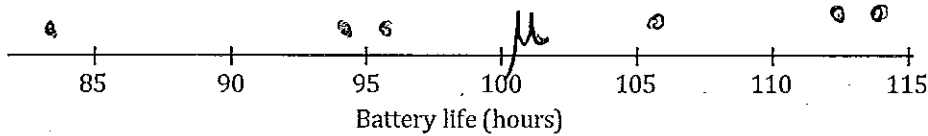


## Lesson 5: Measuring Variability for Symmetrical Distributions

### Classwork

#### Example 1: Calculating the Standard Deviation

Here is a dot plot of the lives of the Brand A batteries from Lesson 4.



How do you measure variability of this data set? One way is by calculating standard deviation.

- First, find each deviation from the mean.
- Then, square the deviations from the mean. For example, when the deviation from the mean is  $-18$ , the squared deviation from the mean is  $(-18)^2 = 324$ .

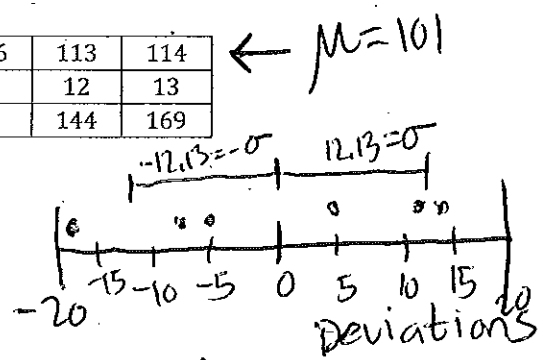
|                                  |       |      |      |     |      |      |
|----------------------------------|-------|------|------|-----|------|------|
| Life (Hours)                     | 83    | 94   | 96   | 106 | 113  | 114  |
| Deviation from the Mean          | $-18$ | $-7$ | $-5$ | $5$ | $12$ | $13$ |
| Squared Deviations from the Mean | 324   | 49   | 25   | 25  | 144  | 169  |

- Add up the squared deviations:

$$324 + 49 + 25 + 25 + 144 + 169 = 736.$$

This result is the *sum* of the squared deviations.

*Too big to matter.*



The number of values in the data set is denoted by  $n$ . In this example,  $n$  is 6.

- You divide the sum of the squared deviations by  $n - 1$ , which here is  $6 - 1$ , or 5.

$$\frac{736}{5} = 147.2$$

*Average of squared deviations?*

- Finally, you take the square root of 147.2, which to the nearest hundredth is 12.13.

*Standard deviation = sigma*

That is the standard deviation! It seems like a very complicated process at first, but you will soon get used to it.

We conclude that a typical deviation of a Brand A battery lifetime from the mean battery lifetime for Brand A is 12.13 hours. The unit of standard deviation is always the same as the unit of the original data set. So, the standard deviation to the nearest hundredth, with the unit, is 12.13 hours. How close is the answer to the typical deviation that you estimated at the beginning of the lesson?

$\sigma \approx 19.6$

Exercises 1–5

Now you can calculate the standard deviation of the lifetimes for the eight Brand B batteries. The mean was 100.5. We already have the deviations from the mean.

|                                 |       |                |       |                |     |      |      |      |
|---------------------------------|-------|----------------|-------|----------------|-----|------|------|------|
|                                 |       | $\mu - \sigma$ | $\mu$ | $\mu + \sigma$ |     |      |      |      |
| Life (Hours)                    | 73    | 76             | 92    | 94             | 110 | 117  | 118  | 124  |
| Deviation from the Mean         | -27.5 | -24.5          | -8.5  | -6.5           | 9.5 | 16.5 | 17.5 | 23.5 |
| Squared Deviation from the Mean |       |                |       |                |     |      |      |      |

1. Write the squared deviations in the table.

$100.5 + 19.6 \approx 120.1$

$100.5 - 19.6 \approx 81$

2. Add up the squared deviations. What result do you get?

$\frac{5}{8}$  are within 1 $\sigma$  of  $\mu$

3. What is the value of  $n$  for this data set? Divide the sum of the squared deviations by  $n - 1$ , and write your answer below. Round your answer to the nearest thousandth.

4. Take the square root to find the standard deviation. Record your answer to the nearest hundredth.

5. How would you interpret the standard deviation that you found in Exercise 4? (Remember to give your answer in the context of this question. Interpret your answer to the nearest hundredth.)

$\frac{2}{3}$  of data is with 1 $\sigma$  of  $\mu$ .  
So about  $\frac{2}{3}$  of batteries have lifetimes between 81 & 120.1 hours.

## Exercises 6–7

Jenna has bought a new hybrid car. Each week for a period of seven weeks, she has noted the fuel efficiency (in miles per gallon) of her car. The results are shown below.

45 44 43 44 45 44 43

6. Calculate the standard deviation of these results to the nearest hundredth. Be sure to show your work.

$$\frac{45 + 44 + 43 + 44 + 45 + 44 + 43}{7} = 44$$

$$\text{Dev: } 1, 0, -1, 0, 1, 0, -1$$

$$\text{Dev}^2: 1, 0, 1, 0, 1, 0, 1$$

$$\text{Total} = 4$$

$$\frac{4}{n-1} = \frac{4}{6} = \frac{2}{3}, \rightarrow \sqrt{\frac{2}{3}} = .82$$

7. What is the meaning of the standard deviation you found in Exercise 6?

That her car will have gas mileage within .82 mpg of 44 on  $\frac{2}{3}$  of her trips.

**Lesson Summary**

- The standard deviation measures a typical deviation from the mean.
- To calculate the standard deviation,
  1. Find the mean of the data set;
  2. Calculate the deviations from the mean;
  3. Square the deviations from the mean;
  4. Add up the squared deviations;
  5. Divide by  $n - 1$  (if working with data from a sample, which is the most common case);
  6. Take the square root.
- The unit of the standard deviation is always the same as the unit of the original data set.
- The larger the standard deviation, the greater the spread (variability) of the data set.

**Problem Set**

1. A small car dealership tests the fuel efficiency of sedans on its lot. It chooses 12 sedans for the test. The fuel efficiency (mpg) values of the cars are given in the table below. Complete the table as directed below.

|                                    |    |    |    |    |    |    |    |    |    |    |    |    |
|------------------------------------|----|----|----|----|----|----|----|----|----|----|----|----|
| Fuel Efficiency (miles per gallon) | 29 | 35 | 24 | 25 | 21 | 21 | 18 | 28 | 31 | 26 | 26 | 22 |
| Deviation from the Mean            |    |    |    |    |    |    |    |    |    |    |    |    |
| Squared Deviation from the Mean    |    |    |    |    |    |    |    |    |    |    |    |    |

- a. Calculate the mean fuel efficiency for these cars.
- b. Calculate the deviations from the mean, and write your answers in the second row of the table.
- c. Square the deviations from the mean, and write the squared deviations in the third row of the table.
- d. Find the sum of the squared deviations.
- e. What is the value of  $n$  for this data set? Divide the sum of the squared deviations by  $n - 1$ .
- f. Take the square root of your answer to part (e) to find the standard deviation of the fuel efficiencies of these cars. Round your answer to the nearest hundredth.