Logarithm Notes

What is a logarithm?
Logarithms are nothing to be scared of. They are just a weird way of saying “the inverse of an exponential function.” Instead of saying “inverse of exponential” over and over, we can just say “log” instead.

Please remember that inverses cancel each other out, regardless of the order. Addition and subtraction are inverses, multiplication and division are inverses, powers and roots are inverses, and exponents and logarithms are inverses.

Here are a few examples:

X + 7 = 10	$\frac{x}{7} = 10$	$\sqrt[3]{x} = 4$	$5^x = 125$
Subtract 7	Multiply by 7	Raise to 3rd power	Log base 5
X = 10 - 7 = 3	X = 10 $\cdot$ 7 = 70	X = 4^3 = 64	X = $log_5 125$ = 3

The reason the answer to $log_5 125$ is 3 is because $5^3 = 125$. The log undid the exponential function.

How do you use a calculator to evaluate logarithms?

You will notice that there is a “log” button on your calculator. Be careful! This button assumes you are doing a “log base 10,” so if you want to do another base, you need to tell the calculator which base you are “logging.”

Some calculators have a shortcut: Math-Alpha-Math. This only works on some calculators.

All calculators can use the change of base formula:

$$log_5 125 = \frac{log(125)}{log(5)} \qquad log_{22} 225 = \frac{log(225)}{log(22)} \qquad log_{753} 1 = \frac{log(1)}{log(753)}$$

How do you use logarithms to solve equations?
Logarithms are the inverse of exponential functions. Logarithms cancel out the base, leaving you with the exponent. If the variable you are solving for is in the exponent, you MUST use a logarithm to solve it algebraically (unless you can figure out the exponent in your head).

Similarly, if the variable is inside of the logarithm, you MUST use an exponent to solve it algebraically.

Here are a few examples: (Read the columns vertically)

Example 1	Example 2	Example 3	Example 4
$3(4)^x - 5 = 10$	$3log_4(x) - 5 = 10$	$7(5)^{x+4} = 175$	$7log_5(x+4) = 14$
$3(4)^x = 15$	$3log_4(x) = 15$	$(5)^{x+4} = 25$	$log_5(x+4) = 2$
$(4)^x = 5$	$log_4(x) = 5$	$log_5(5)^{x+4} = log_5 25$	$5^{log_5(x+4)} = 5^2$
$log_4(4)^x = log_4 5$	$4^{log_4(x)} = 4^5$	$x + 4 = log_5 25$	$x + 4 = 25$
$x = log_4 5$	$x = 1024$	$x + 4 = \frac{log(25)}{log(5)} = 2$	$x = 21$
$x = \frac{log(5)}{log(4)} = 1.16$		$x = -2$	

Please notice that you always do the same thing to both sides. If both sides were exponential, you used a logarithm with the SAME BASE to cancel out the exponential. If both sides were logarithmic, you make both sides an exponent with the SAME BASE to cancel the logarithm. Logs and exponents are inverses. Logs cancel bases.

Inverses and Logarithms

Everything we know about logarithms comes from exponential functions and inverses.

Remember that inverses switch inputs and outputs. Because inputs and outputs are switched, the order of operations is also switched (remember PEMDAS and SADMEP?). This means that anything that was INSIDE the parentheses for the function will be OUTSIDE the parentheses for the inverse.

$y = 11log_5(x-5) + 3$ ------------------------------------->>> $y^{-1} = 5^{\frac{x-3}{11}} + 5$

List of operations:

1. Minus 5
2. Log base 5
3. Times 11
4. Plus 3

Inverse List:

1. Minus 3
2. Divide 11
3. Exponent base 5
4. Plus 5

$y = 8(3)^{3x-4}$ ------------------------------------->>> $y^{-1} = (log_3(\frac{x}{8}) + 4) \div 3$

List of operations:

1. Times 3
2. Minus 4
3. Exponent base 3
4. Times 8

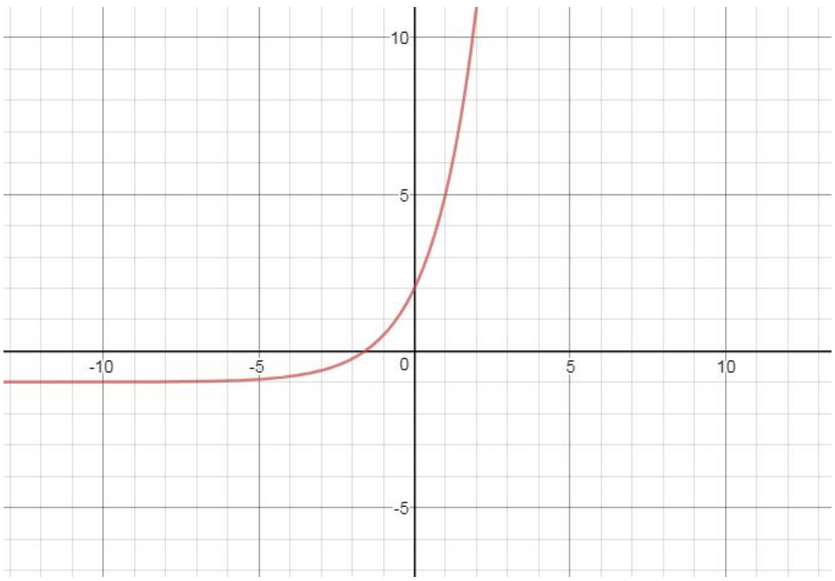
Inverse List:

1. Divide 8
2. Log base 3
3. Plus 4
4. Divide 3

Graphs of Logarithms:

Recall that the graph of an inverse is a reflection over the line y=x (because the inputs and outputs are switched.) Recall also that you can switch the x,y coordinates of each point to create the graph of an inverse.

This is the graph of $y = 3(2)^x - 1$
The **horizontal asymptote** is y = -1.
The **Initial value** is 3 steps above the **horizontal asymptote,** so y = 2.
The **Growth Factor** is 2, which means the y values are twice as high above the **horizontal asymptote** for every step you take to the right.



Everything is inverted for the logarithm: $y = log_2(\frac{x+1}{3})$
Vertical Asymptote x = -1. Growth factor of 2 compared to asymptote.