

Mr. Maurer is racing a student in the 100 meter dash, and he gives the student a head start. Maurer knows that the winner of the race depends on how much of a head start he gives the student. He decides to run the race 2 times to figure out the perfect head start. Each race has a different head start. The runners always run their same constant speed in each race. For every table, x stands for the time (in seconds), and y stands for the distance (in meters).

1. First Race:

x	0	1	2	3	6	7	8
Y (Maurer)	0	12	24	36	72	84	96
Y (Student)	40	48	56	64	88	96	104

Handwritten notes: Arrows show +12 for Maurer and +8 for Student. A bracket under the first two columns shows -8.

- How can you use the table to find the head start? What vocab word is the head start?
Find the difference in Y-values and work backwards to zero. The head start is the y-intercept.
- How can you use the table to find the speed of each runner? What vocab word is the speed?
Find the difference in Y-values & divide by the difference in x-values. The speed is the slope.
- Who wins the 100 meter dash?
*Maurer: 72-36=36, 6-3=3, 36÷3=12
 Student: 88-64=24, 6-3=3, 24÷3=8
 The student, between 7 & 8 seconds.*

d. If they kept running, when will they meet up?

Handwritten work for problem d:

$$\begin{array}{l}
 \text{Maurer (M)} \\
 y = 12x + 0 \\
 \text{Student (S)} \\
 y = 8x + 40
 \end{array}
 \quad
 \begin{array}{l}
 M = S \\
 12x = 8x + 40 \\
 -8x \quad -8x \\
 \hline
 4x = 40 \\
 \frac{4x}{4} = \frac{40}{4} \\
 x = 10
 \end{array}
 \quad
 \begin{array}{l}
 \text{M} \\
 y = 12(10) = 120 \\
 \text{S} \\
 y = 8(10) + 40 = 80 + 40 = 120 \\
 \text{120 meters}
 \end{array}$$

Answers: 10 seconds, 120 meters.

2. Second Race:

x	0	1	2	3	7	8	9
Y (Maurer)	0	12	24	36	84	96	108
Y (Student)	30	38	46	54	86	94	102

- What is the head start?
M=0, S=30
- What is each runner's speed?
M=12, S=8
- Who wins?
Maurer. He is ahead between 7 & 8 seconds.
- When do they meet up?

Handwritten work for problem d:

$$\begin{array}{l}
 \text{Maurer (M)} \\
 y = 12x + 0 \\
 \text{Student (S)} \\
 y = 8x + 30
 \end{array}
 \quad
 \begin{array}{l}
 M = S \\
 12x = 8x + 30 \\
 -8x \quad -8x \\
 \hline
 4x = 30 \\
 \frac{4x}{4} = \frac{30}{4} \\
 x = 7.5
 \end{array}
 \quad
 \begin{array}{l}
 \text{M} \\
 y = 12(7.5) = 90 \\
 \text{S} \\
 y = 8(7.5) + 30 = 60 + 30 = 90 \\
 \text{90 meters}
 \end{array}$$

Answers: 7.5 seconds, 90 meters.

3. Mr. Maurer claims that he can figure out the perfect head start from the results of the first two races. A perfect head start would mean that the two runners tie (they finish the 100 meter race in exactly the same amount of time).

Use tables, graphs, and/or equations to find the perfect head start. Remember that each runner runs at the same speed as in the first two races.

Table Method: Know the speeds, work backwards from 100 meter tie.

$X = \text{Time}$									
$Y_1 = \text{Maurer Distance}$	100	88	76	64	52	40	28	16	4
$Y_2 = \text{Student Distance}$	100	92	84	76	68	60	52	44	36

I don't know the times yet. I'm left over with 4 meters for Maurer's Distance. Speed is 12 mps. So using $D = rt$, we know $t = \frac{D}{r} = \frac{4}{12} = \frac{1}{3}$. Now I will use the time $\frac{1}{3}$ second, to find the student distance.

$D = 8 \cdot \frac{1}{3} = 2\frac{2}{3}$. Then redo the table, by subtracting D from 36

$X = \text{Time}$	0	1	2	3	4	5	6	7	8	$8\frac{1}{3}$
$Y_1 = \text{Maurer}$	0	12	24	36	48	60	72	84	96	100
$Y_2 = \text{Student}$	$33\frac{1}{3}$	$41\frac{1}{3}$	$49\frac{1}{3}$	$57\frac{1}{3}$	$65\frac{1}{3}$	$73\frac{1}{3}$	$81\frac{1}{3}$	$89\frac{1}{3}$	$97\frac{1}{3}$	100

EQ Method: Know the speeds, distance. Plug in 100.

M
 $y = 12x$
 $\frac{100}{12} = \frac{12x}{12}$
 $8\frac{1}{3} = x$

S
 $y = 8x + b$
~~100~~ $100 = 8(8\frac{1}{3}) + b$
 $100 = 66\frac{2}{3} + b$
 $-66\frac{2}{3} \quad -66\frac{2}{3}$
 $33\frac{1}{3} = b$

Don't know the head start yet.

The perfect head start is $33\frac{1}{3}$ meters. They will tie after $8\frac{1}{3}$ seconds at the 100 meter line.