Multiplying Functions.

One of the most important features of a graph is where the roots are.

The zero product property says that if your product is zero, one of your factors must be zero.

If you multiply functions, then the ZPP lets you look at the roots of each function separately.

So if you want to know the roots of $f(x) \cdot g(x)$, just identify the roots of each function separately.

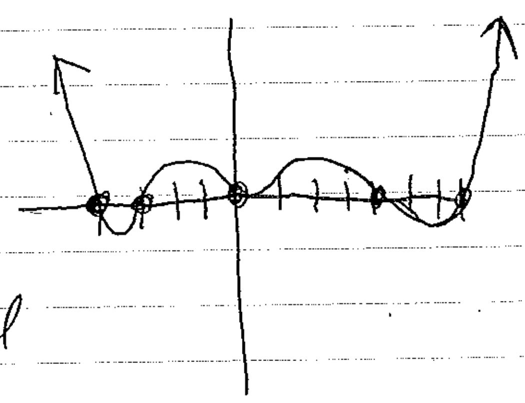
Examples: $f(x) = x^3 - 4x^2 - 21x = x(x-7)(x+3)$
$g(x) = x^3 - 16x = x(x+4)(x-4)$

So $f(x)$ had roots of $x=0, 7, -3$
& $g(x)$ had roots of $x=0, -4, 4$.

Thus $f(x) \cdot g(x)$ has all of those, $x = 0, -3, -4, 4, 7$.

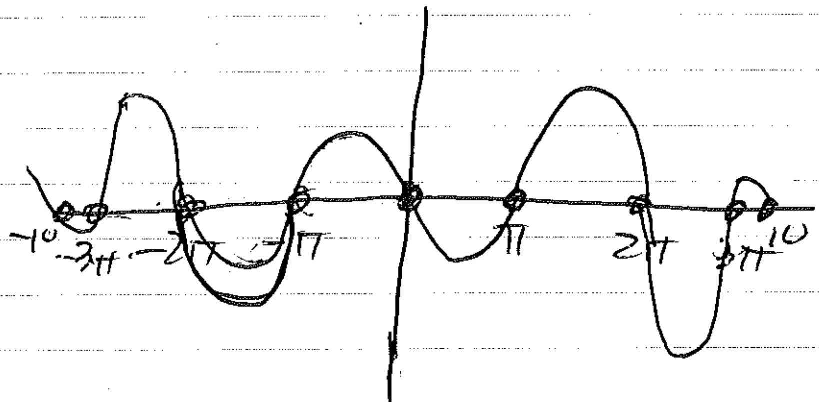
& $f(x) \cdot g(x) = x^2(x-7)(x+3)(x+4)(x-4)$

Notice how x was a factor of both f&g, so now it is a repeated root, or a bounce.



Ex 2) $f(x) = \sin x \leftarrow$ roots of $0, \pm\pi, 2\pi, 3\pi$
$g(x) = 2x^2 - 200 = 2(x^2 - 100) = 2(x-10)(x+10)$

So $f(x) \cdot g(x) = \sin x \cdot (2x^2 - 200)$ has roots of $0, \pm\pi, \pm 2\pi, \pm 3\pi, \pm 10, \pm 4\pi$ etc



To tell if it is above or below the x-axis, just plug in values in each interval.

For example, between π & 2π I can plug in any number within that range. I'll use $\frac{3\pi}{2}$ (because thats on the unit circle)

$$\sin \frac{3\pi}{2} = -1, \quad 2\left(\frac{3\pi}{2}\right)^2 - 200 < 0$$

So $f(x)$ is negative, so is $g(x)$. Thus

$f(x) \cdot g(x)$ is positive from $(\pi, 2\pi)$

Exercises:

1) $f(x) = 3x^3 - 15x^2 - 42x$
$g(x) = 16x^2 - 9$

2) $f(x) = \cos x$
3) $g(x) = \cos(2x)$