CCSS Advanced Algebra 4 Operations with Complex Numbers Name _____

1. Use an area model to show that the function $g(x) = (x - 2 - i)(x - 2 + i)$ is equivalent to $g(x) = x^2 - 4x + 5$ in Standard Form.

REMEBER: $i^2 = -1$

	x	-2	$+i$
x	x^2	$-2x$	$+ix$
-2	$-2x$	4	$-2i$
$-i$	$-ix$	$+2i$	$-i^2$

$= x^2 - 2x - 2x + ix - ix + 4 + 2i - 2i - i^2$

$= x^2 - 4x + 4 - i^2 = x^2 - 4x + 4 - (-1)$

$= x^2 - 4x + 5$

2. *Addition and Subtraction of Complex Numbers (think like terms…)*
Simplify each sum or difference to the form $a + bi$.

a. $(3 + i) + (2i - 1)$

$2 + 3i$

b. $(3i - 4) - (5 - 2i)$

$-9 + 5i$

c. $(i^2 + 2i + 1) - (3i - 5)$

$(-1 + 2i + 1) - (3i - 5)$

$(2i) - (3i - 5)$

$5 - i$

3. *Complex Equations.* Check your answers.

a. Solve $w + (6 + i) = 3$ for w

$-(6+i) \quad -(6+i)$

$w = 3 - (6 + i)$

$w = -3 - i$

b. Solve $3w - 2i = w + 4i - 6$ for w

$-w \quad -w$

$2w - 2i = 4i - 6$

$+2i \quad +2i$

$\frac{2w}{2} = \frac{6i - 6}{2} \rightarrow w = 3i - 3$

4. Two Complex Numbers are called **Conjugates** if they are in the form $a + bi$ and $a - bi$.

a. Which of the following pairs of complex numbers are conjugates? Select all that apply.

~~$3 + 2i$ and $-3 + 2i$~~ ~~$3 + 2i$ and $-3 - 2i$~~ i and $-i$ $3 + 2i$ and $3 - 2i$

b. What happens when you add conjugates? In other words, what is $(a + bi) + (a - bi)$?

$(a+bi)+(a-bi) = 2a$

c. What happens when you subtract conjugates? In other words, what is $(a + bi) - (a - bi)$?

$(a+bi)-(a-bi) = 2bi$

4. *Multiplication of Complex Numbers*:
Use an Area Model to complete each product. Write the answer in the form $a + bi$.

a. $(3 + i)(2i - 1)$

	3	$+i$
$2i$	$6i$	$2i^2$
-1	-3	$-i$

$= 5i - 3 + 2i^2 = 5i - 3 - 2 = -5 + 5i$

b. $(3i - 4)(5 - 2i)$

	$3i$	-4
5	$15i$	-20
$-2i$	$-6i^2$	$8i$

$= 23i - 20 - 6i^2$

$= 23i - 20 - 6(-1)$

$= 23i - 20 + 6$

$-14 + 23i$

c. $i(2i - 5)$

$= 2i^2 - 5i$

$= 2(-1) - 5i$

$= -2 - 5i$

d. $(-i + 5)(-i - 5)$

	$-i$	$+5$
$-i$	i^2	$-5i$
-5	$5i$	-25

$= i^2 - 25 = -1 - 25 = -26$

e. $(4 + 2i)(4 - 2i)$

	4	$+2i$
4	16	$8i$
$-2i$	$-8i$	$-4i^2$

$= 16 - 4i^2$

$16 - 4(-1)$

20

5. Given your answer to parts (d) and (e), what is the product of Conjugate Complex Numbers? In other words, what is $(a+bi)(a-bi)$ for any values of a and b?

	a	$+bi$
a	a^2	abi
$-bi$	$-abi$	$-b^2i^2$

$= a^2 + abi - abi - b^2i^2$

$= a^2 - b^2(-1) = a^2 + b^2$

6. a. Find the roots of $f(x) = 4x^2 + 9$ and show they are Conjugate Complex Numbers.

$4x^2 + 9 = 0$

$-9 \quad -9$

$\frac{4x^2}{4} = \frac{-9}{4}$

$x^2 = \sqrt{-\frac{9}{4}}$

$x = \frac{3}{2}i \qquad x = -\frac{3}{2}i$

b. Find the roots of $g(x) = x^2 + 2x + 3$ and show they are Conjugate Complex Numbers.

$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(3)}}{2(1)}$

$= \frac{-2 \pm \sqrt{4-12}}{2}$

$\frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm \sqrt{8}i}{2}$

c. Use the Quadratic Formula to explain why the complex roots of $y = ax^2 + bx + c$ must be conjugates.

Because of the "±" before the √. If √−# then you +& − an imaginary #. Thus, your roots are conjugates.

7. A polynomial has roots $x = 1$, $x = 2$, $x = 4 - i$ *and* $x = 4 + i$. Write the polynomial in Standard Form.

$(x-1)(x-2)(x-(4-i))(x-(4+i))$

$(x^2 - 3x + 2)(x - 4 + i)(x - 4 - i)$

$(x^2 - 3x + 2)(x^2 - 8x + 17)$

$= x^4 - 11x^3 + 43x^2 - 73x + 34$

	x^2	$-8x$	17
x	x^4	$-8x^3$	$17x^2$
$-3x$	$-3x^3$	$24x^2$	$-51x$
2	$2x^2$	$-16x$	34

	x	-4	$+i$
x	x^2	$-4x$	ix
-4	$-4x$	16	$-4i$
$-i$	$-ix$	$4i$	$-i^2$

8. Challenge: Solve $w(1-i) = 5 - i$ Solve for w.

$w = a + bi$

$(a+bi)(1-i) = 5 - i$

$a - ai + bi - bi^2 = 5 - i$

$a + b = 5$

$-a + b = -1$

$2b = 4$

$b = 2 \quad a = 3 \rightarrow w = 3 + 2i$

9. Practice Rational Expressions:

Simplify each of the following:

a. $\frac{2x^2}{x+1} - \frac{2}{x+1} = \frac{2x^2 - 2}{x+1}$

$= \frac{2(x^2-1)}{x+1} = \frac{2(x+1)(x-1)}{x+1}$

$= 2(x-1)$

b. $\frac{3}{x-2} + \frac{1}{x+3}$

$\frac{3}{x-2}\left(\frac{x+3}{x+3}\right) + \frac{1}{x+3}\left(\frac{x-2}{x-2}\right)$

$\frac{3x+9+x-2}{(x-2)(x+3)} = \frac{4x+7}{(x-2)(x+3)}$

c. $\frac{4}{(x-2)(x+2)} - \frac{1}{x-2}\left(\frac{x+2}{x+2}\right)$

$\frac{4 - x - 2}{(x-2)(x+2)} = \frac{2-x}{(x-2)(x+2)}$

$= \frac{-(x-2)}{(x-2)(x+2)} = \frac{-1}{x+2}$

d. $\frac{5}{x} + \frac{x}{x^2+x}$

$\frac{5}{x} + \frac{x}{x(x+1)} = \frac{5}{x} + \frac{1}{x+1}$

$= \frac{5}{x}\left(\frac{x+1}{x+1}\right) + \frac{1}{x+1} \cdot \frac{x}{x}$

$= \frac{5x+5+x}{x(x+1)} = \frac{6x+5}{x(x+1)}$

e. $\frac{(x-3)(x+4)}{(x-1)^2} \cdot \frac{(x-1)}{(x-3)(x-4)^2}$

$= \frac{x+4}{(x-1)(x-4)^2}$

f. $\frac{x^2+2x+1}{x^2-25} \cdot \frac{x^2-6x+5}{x^2-1}$

$\frac{(x+1)(x-5)(x-1)}{(x-5)(x+5)(x+1)(x-1)}$

$\frac{x+1}{x+5}$