CCSS Advanced Algebra 4 Operations with Complex Numbers Name ____________________

1. Use an area model to show that the function $g(x) = (x-2-i)(x-2+i)$ is equivalent to $g(x) = x^2 - 4x + 5$ in Standard Form.

	x	-2	$-i$
x	x^2	$-2x$	$-ix$
-2	$-2x$	4	$2i$
i	xi	$-2i$	$-i^2$

$= x^2 - 4x + 4 - i^2$

$x^2 - 4x + 4 + 1 = x^2 - 4x + 5$

2. *Addition and Subtraction of Complex Numbers (think like terms…)*
Simplify each sum or difference to the form $a + bi$.

a. $(3+i)+(2i-1)$

$2 + 3i$

b. $(3i-4)-(5-2i)$

$3i - 4 - 5 + 2i$

$5i - 9$

c. $(i^2+2i+1)-(3i-5)$

$(-1+2i+1)$

$2i - 3i + 5$

$-i + 5$

3. *Complex Equations*. Check your answers.

a. Solve $w+(6+i)=3$ for w

$w = -3 - i$

Check: $-3 - i + 6 + i$

$-3 + 6 = 3$

b. Solve $3w - 2i = w + 4i - 6$ for w

$+2i \quad +2i$

$3w = w + 6i - 6$

$-w \quad -w$

$2w = 6i - 6$

$w = 3i - 3$

4. Two Complex Numbers are called **<u>Conjugates</u>** if they are in the form $a + bi$ and $a - bi$.

a. Which of the following pairs of complex numbers are conjugates? Select all that apply.

$3+2i$ *and* $-3+2i$ $\qquad$ $3+2i$ *and* $-3-2i$ $\qquad$ i *and* $-i$ $\qquad$ $3+2i$ *and* $3-2i$

b. What happens when you add conjugates? In other words, what is $(a+bi)+(a-bi)$?

$a + bi$

$+\ a - bi$

$= 2a$

c. What happens when you subtract conjugates? In other words, what is $(a+bi)-(a-bi)$?

$a + bi$

$-(a - bi) = bi - (-bi)$

$2bi$

4. *Multiplication of Complex Numbers*:
Use an Area Model to complete each product. Write the answer in the form $a + bi$.

a. $(3+i)(2i-1)$

	3	$+i$
$2i$	$6i$	$2i^2$
-1	-3	$-i$

$= 5i - 3 + 2i^2$

$5i - 3 - 2$

$5i - 5$

$-5 + 5i$

b. $(3i-4)(5-2i)$

	$3i$	-4
5	$15i$	-20
$-2i$	$-6i^2$	$8i$

$= 23i - 20 - 6i^2$

$23i - 20 + 6$

$23i - 14$

$-14 + 23i$

c. $i(2i-5) = 2i^2 - 5i$

$-2 - 5i$

d. $(-i+5)(-i-5)$

	$-i$	$+5$
$-i$	i^2	$-5i$
-5	$5i$	-25

$= i^2 - 25$

$-1 - 25 = -26$

e. $(4+2i)(4-2i)$

	4	$2i$
4	16	$8i$
$-2i$	$-8i$	$-4i^2$

$= 16 - 4i^2$

$16 + 4 = 20$

5. Given your answer to parts (d) and (e), what is the product of Conjugate Complex Numbers? In other words, what is $(a+bi)(a-bi)$ for any values of a and b?

	a	$+bi$
a	a^2	abi
$-bi$	$-abi$	$-b^2i^2$

$= a^2 - b^2i^2$

$a^2 + b^2$

6. a. Find the roots of $f(x) = 4x^2 + 9$ and show they are Conjugate Complex Numbers.

$4x^2+9=0$

$4x^2=-9$

$x^2=-\frac{9}{4}$

$x = \pm\sqrt{-9/4}$

$x = \pm\frac{3}{2}i$

$0+\frac{3}{2}i$

$0-\frac{3}{2}i$

b. Find the roots of $g(x) = x^2 + 2x + 3$ and show they are Conjugate Complex Numbers.

$x = \frac{-2\pm\sqrt{4-4(1)(3)}}{2(1)}$

$= \frac{-2\pm\sqrt{4-12}}{2}$

$\frac{-2\pm\sqrt{-8}}{2}$

$-\frac{2}{2}\pm\frac{\sqrt{8}i}{2}$

$-1\pm\sqrt{2}i$

$-1+\sqrt{2}i$

$-1-\sqrt{2}i$

c. Use the Quadratic Formula to explain why the complex roots of $y = ax^2 + bx + c$ must be conjugates.

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

The + & − give conjugates

7. A polynomial has roots $x=1,\ x=2,\ x=4-i$ and $x=4+i$. Write the polynomial in Standard Form.

$(x-1)(x-2)(x-(4-i))(x-(4+i))$

$(x^2-3x+2)(x^2-8x+17)$

$x^4-8x^3+17x^2-3x^3+24x^2-51x+2x^2-16x+34$

$x^4-11x^3+43x^2-67x+34$

8. Challenge: Solve $w(1-i) = 5-i$ Solve for w.

$w = \frac{5-i}{1-i}\left(\frac{1+i}{1+i}\right) = \frac{5+5i-i-i^2}{1+1} = \frac{4i+6}{2} = 2i+3$

9. Practice Rational Expressions:

Simplify each of the following:

a. $\frac{2x^2}{x+1} - \frac{2}{x+1} = \frac{2x^2-2}{x+1}$

$\frac{2(x^2-1)}{x+1} = \frac{2\cancel{(x+1)}(x-1)}{\cancel{x+1}}$

$2(x-1)$

b. $\frac{3}{x-2} + \frac{1}{x+3} = \frac{3(x+3)}{(x-2)(x+3)} + \frac{1}{x+3}\frac{(x-2)}{(x-2)}$

$\frac{3x+9+x-2}{(x-2)(x+3)} = \frac{4x+7}{(x-2)(x+3)}$

c. $\frac{4}{(x-2)(x+2)} - \frac{1}{x-2}\left(\frac{x+2}{x+2}\right)$

$\frac{4-x-2}{(x-2)(x+2)} = \frac{2-x}{(x-2)(x+2)}$

$= -\frac{1}{x+2}$

d. $\frac{5}{x} + \frac{x}{x^2+x}$

$\frac{(x+1)}{(x+1)}\frac{5}{x} + \frac{1}{x+1}\left(\frac{x}{x}\right) = \frac{5x+5+x}{(x+1)x}$

$\frac{6x+5}{(x+1)x}$

e. $\frac{\cancel{(x-3)}(x+4)}{(x-1)^2} \cdot \frac{\cancel{(x-1)}}{\cancel{(x-3)}(x-4)^2}$

$\frac{x+4}{(x-1)(x-4)^2}$

f. $\frac{x^2+2x+1}{x^2-25} \cdot \frac{x^2-6x+5}{x^2-1}$

$\frac{(x+1)^2\cancel{(x-5)}\cancel{(x-1)}}{\cancel{(x-5)}(x+5)\cancel{(x+1)}\cancel{(x-1)}}$

$\frac{x+1}{x+5}$