Operations with Functions Review Packet

C LEVEL:

1. Let $f(x) = x^2(x - 7)(x + 3)^3$ and $g(x) = x^2(x + 5)(x - 7)$

   a. Find and simplify $\frac{f(x)}{g(x)}$

   $\frac{x^2(x-7)(x+3)^3}{x^2(x+5)(x-7)} = \frac{(x+3)^3}{(x+5)}$

   b. What is the domain and range of $\frac{f(x)}{g(x)}$?

   D: $x \neq -5, 0, 7$ R: $(-\infty, \infty)$

   c. What is the end behavior (both directions) of $\frac{f(x)}{g(x)}$?

   $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)} = \infty$ Top > Bottom, either $\pm\infty$.

   d. What are the roots of $\frac{f(x)}{g(x)}$?

   $x = -3$

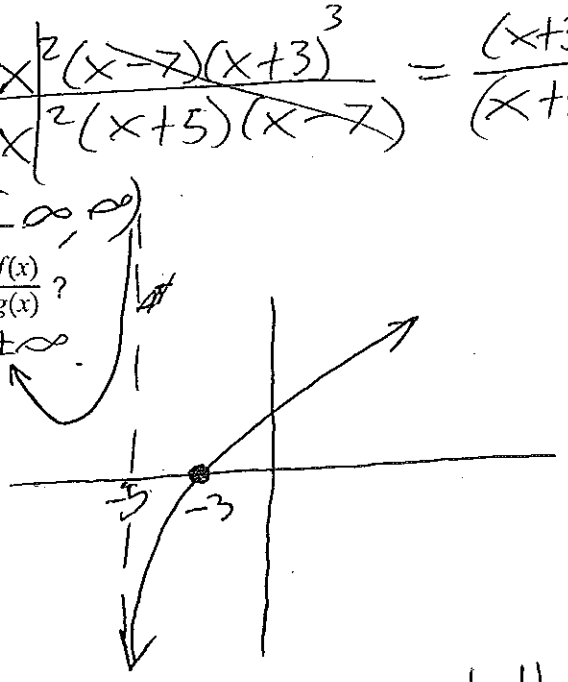
   e. What are the holes of $\frac{f(x)}{g(x)}$?

   $x = 7$ & 0

   f. What are the asymptotes of $\frac{f(x)}{g(x)}$?

   $x = -5$

   g. Sketch a graph of $\frac{f(x)}{g(x)}$.

   pos, neg, pos
   $(-)^3$ $(-)^3$
   $(-)$ -5 $(+)$ -3

   ← Because both "ends" of the # line are positive end behavior is positive ∞

2. Let $p(x) = 3x(x + 10)(x - 4)^2$ and $q(x) = (x - 4)(x - 5)(x - 6)$

   a. Find and simplify $\frac{p(x)}{q(x)} = \frac{3x(x+10)(x-4)^2}{(x-4)(x-5)(x-6)} = \frac{3x(x+10)(x-4)}{(x-5)(x-6)}$

   b. What is the domain and range of $\frac{p(x)}{q(x)}$?

   D: $x \neq 4, 5, 6$ R: $(-\infty, \infty)$

   c. What is the end behavior (both directions) of $\frac{p(x)}{q(x)}$?

   Top > Bottom

   d. What are the roots of $\frac{p(x)}{q(x)}$?

   4 will act like a root (really) but it's a hole $x = 0, x = -10$

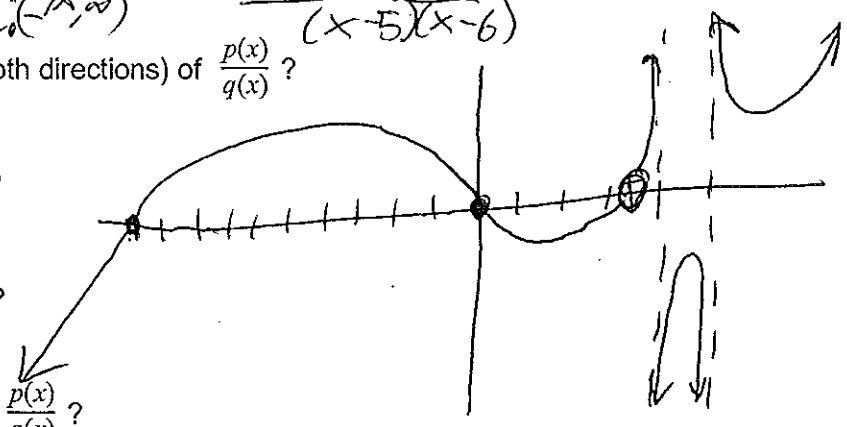
   e. What are the holes of $\frac{p(x)}{q(x)}$?

   $x = 4$

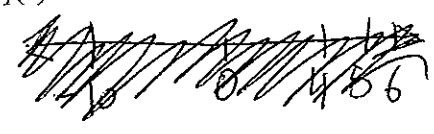
   f. What are the asymptotes of $\frac{p(x)}{q(x)}$?

   $x = 5, x = 6$

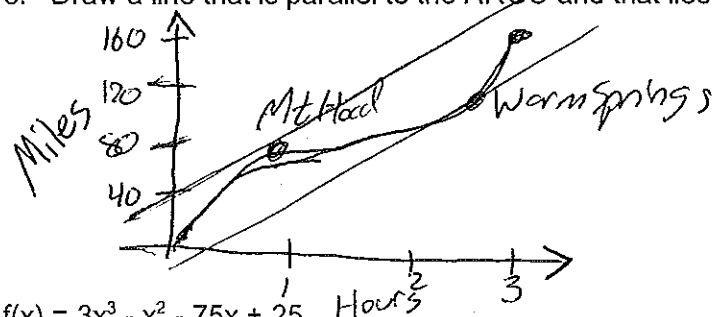
   g. Sketch a graph of $\frac{p(x)}{q(x)}$.

   neg, pos, neg, pos, neg, pos.
   $(-)(-)(-)$ $(-)(+)(-)$ $(+)(+)(-)$ $(+)(+)(+)$
   $(-)(-)$ -10 $(-)(-)$ 0 $(+)(-)$ 4 $(-)(+)$ 5 $(+)(-)$ 6



3. Mr. Maurer is driving from Portland to Bend (160 miles from PDX) and the trip takes him 3 hours. On his way he drives over Mt. Hood (50 miles from PDX) and the curves in the mountains make him slow down. Once he reaches Warm Springs (100 miles from PDX) the road straightens out and there is no traffic.
   a. What is his AROC? $\frac{160}{3} = 53\frac{1}{3}$ mph
   b. Does he always drive at that speed? No
   c. Does he ever drive at exactly that speed? Yes
   d. Draw a possible graph of Mr. Maurer's trip to Bend. Please label your axes and any important points.
   e. Draw a line that is parallel to the AROC and that lies tangent to your graph.

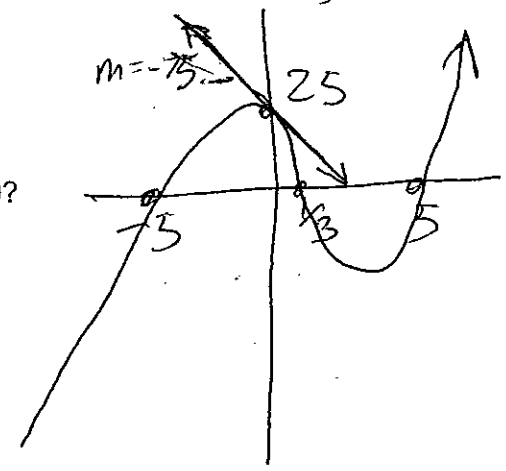


4. Let $f(x) = 3x^3 - x^2 - 75x + 25$
   a. What are the roots of f(x)? $x = \frac{1}{3}, -5, 5$ 5 is a root.
   b. Sketch a graph of f(x). Label roots and y-intercept.
   c. Fill in the following table

$$x-5\overline{)3x^3-x^2-75x+25}$$
$$3x^2+14x-5$$
$$-(3x^3-15x^2)$$
$$14x^2-75x$$
$$-(14x^2-70x)$$
$$-5x+25$$
$$-(-5x+25)$$
$$0$$

$$(3x^2+14x-5)(x-5)$$
$$(3x-1)(x+5)(x-5)$$

x_1	x_2	$\Delta y = y_2 - y_1$	$\frac{\Delta y}{\Delta x}$
0	1	$-48-25$ -73	-73
0	-1	$96-25$ -71	-71
0	0.1	$12.493-25$	-75.07
0	0.01	$24.25-25$ $-.75$	$\frac{.75}{.01} = 75$
0	0.001	$24.925-25$ $-.075$	-75



   d. What is the instantaneous rate of change at x=0?
   e. Draw the tangent line at x=0 -75

x^2+x-6

b. Find and simplify $\frac{p(x)}{q(x)}$

$\frac{(x^2-25)}{x^2+25} \div \frac{(x+3)(x-2)}{x^2+9}$

i. What is the domain and range of $\frac{p(x)}{q(x)}$?

ii. What is the end behavior (both directions) of $\frac{p(x)}{q(x)}$? $\lim = 1$

$= \frac{(x^2-25)(x^2+9)}{(x^2+25)(x+3)(x-2)}$

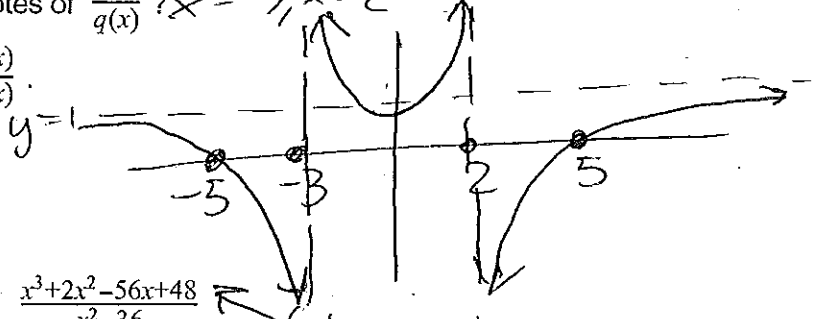
iii. What are the roots of $\frac{p(x)}{q(x)}$? $x = 5, -5$

iv. What are the holes of $\frac{p(x)}{q(x)}$? None

$\frac{x^4-16x^2-225}{x^4+x^3+19x^2+125x-150}$

v. What are the asymptotes of $\frac{p(x)}{q(x)}$? $x=-3, x=2$

vi. Sketch a graph of $\frac{p(x)}{q(x)}$.



2. Let p(x) = $\frac{x^4-5x^3-16x^2+21x-9}{x^3-9x}$ and q(x) = $\frac{x^3+2x^2-56x+48}{x^2-36}$

x^4-8x^2-9

6 is a root.

a. Find and simplify p(x) + q(x)

$p(x) = \frac{x^4-8x^2-9}{x^3-9x}$

$\frac{(x^2-9)(x^2+1)}{x(x^2-9)}$

$\frac{x^2+1}{x}$

$x-6 \overline{)\, x^3+2x^2-56x+48}$

x^2+8x-8

$-(x^3-6x^2)$

$8x^2-56x$

$-(8x^2-48x)$

$-8x+48$

$q(x) = \frac{(x-6)(x^2+8x-8)}{(x+6)(x-6)}$

i. What is the domain and range of p(x) + q(x)?

ii. What is the end behavior (both directions) of p(x) + q(x)?

iii. What are the roots of p(x) + q(x)? -7.52

iv. What are the holes of p(x) + q(x)? Used calc.

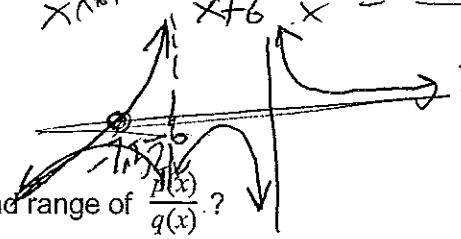
v. What are the asymptotes of p(x) + q(x)? $0, -6$

vi. Sketch a graph of p(x) + q(x).

$p(x)+q(x) = \frac{x^2+1}{x} + \frac{x^2+8x-8}{x+6} \cdot \frac{x}{x} = \frac{(x^2+1)(x+6)+(x^2+8x-8)x}{(x+6)\cdot x}$

$\frac{x^3+6x^2+x+6+x^3+8x^2-8x}{x^2+6x}$

$\frac{2x^3+14x^2-7x+6}{x^2+6x}$



b. Find and simplify $\frac{p(x)}{q(x)}$

i. What is the domain and range of $\frac{p(x)}{q(x)}$?

ii. What is the end behavior (both directions) of $\frac{p(x)}{q(x)}$? $\lim = 1$

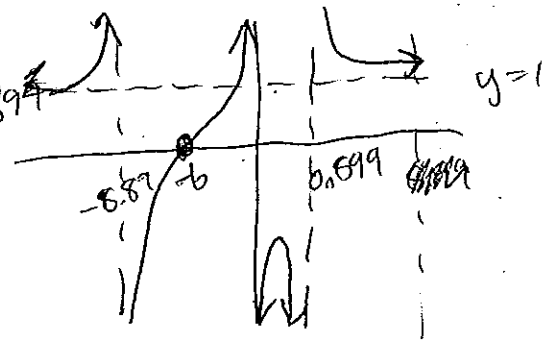
iii. What are the roots of $\frac{p(x)}{q(x)}$? $x=-6$

iv. What are the holes of $\frac{p(x)}{q(x)}$? $x=3, -3, 6$

v. What are the asymptotes of $\frac{p(x)}{q(x)}$? $x=0, -8.89, 0.89$

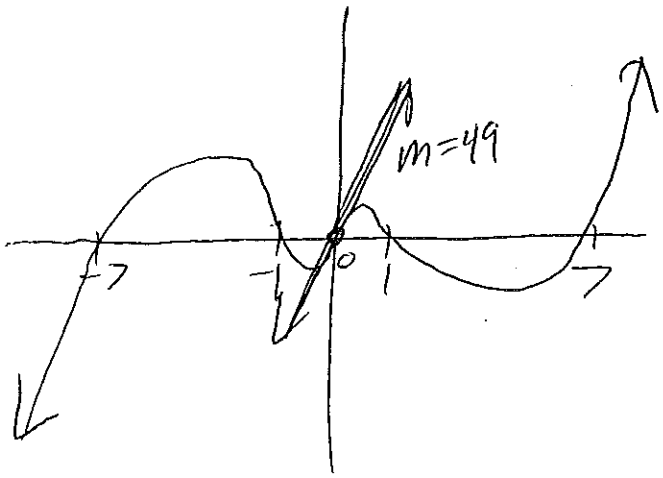
vi. Sketch a graph of $\frac{p(x)}{q(x)}$.

$\frac{x^2+1}{x} \div \frac{x^2+8x-8}{x+6} = \frac{x^2+1}{x} \cdot \frac{x+6}{x^2+8x-8}$



5. Let $g(x) = x^5 - 50x^3 + 49x = x(x^4 - 50x^2 + 49) = x(x^2-49)(x^2-1)$
   a. What are the roots of g(x)? $x = \pm 7, \pm 1, 0$ $= x(x+7)(x-7)(x+1)(x-1)$
   b. Sketch a graph of g(x). Label roots and y-intercept.
   c. Fill in the following table

x_1	x_2	$\Delta y = y_2 - y_1$	$\frac{\Delta y}{\Delta x}$
0	1	0−0 = 0	0
0	-1	0−0 = 0	0
0	0.1	4.85	48.5
0	0.01	.48995	48.995
0	0.001	.049	49



   d. What is the instantaneous rate of change at x=0? 49
   e. Draw the tangent line at x=0

AB LEVEL

roots of $-3, 2$, $(x+3)(x-2) = x^2+x-6$

1. Let $p(x) = \frac{x^4-22x^2-75}{x^4+28x^2+75}$ and $q(x) = \frac{x^4+x^3-2x^2+4x-24}{x^4+13x^2+36}$

$p(x) = \frac{(x^2-25)(x^2+3)}{(x^2+25)(x^2+3)}$

$q(x) = \frac{(x+3)(x-2)(x^2+4)}{(x^2+9)(x^2+4)}$

   a. Find and simplify p(x) + q(x)
      i. What is the domain and range of p(x) + q(x)? D: ∞ R: ∞
      ii. What is the end behavior (both directions) of p(x) + q(x)?
      iii. What are the roots of p(x) + q(x)?
      iv. What are the holes of p(x) + q(x)? None
      v. What are the asymptotes of p(x) + q(x)? None
      vi. Sketch a graph of p(x) + q(x).

$x^2+x-6 \overline{)\, x^4+x^3-2x^2+4x-24}$

$x^2 + 4$

$-(x^4+x^3-6x^2)$

$4x^2$

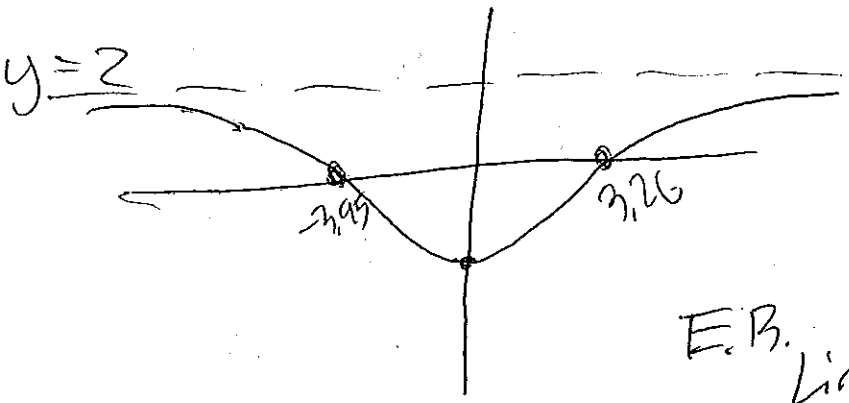
$p(x)+q(x) = \frac{x^2-25}{x^2+25} + \frac{(x+3)(x-2)}{x^2+9}$

$\frac{(x^2-25)(x^2+9) + (x+3)(x-2)(x^2+4)}{(x^2+25)(x^2+9)}$

$\frac{x^4-16x^2-225+x^4+x^3+19x^2+25x-150}{x^4+34x^2+225}$

$\frac{2x^4+3x^2+x^3+25x-375}{x^4+34x^2+225}$

From Calc, roots $x = -3.95$, $x = 3.26$



E.B. $\lim = 2$

Remember that instantaneous rate of change is defined: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

1. Use the definition to find the IROC of the following functions for any x.
   a. f(x) = 3x $\frac{3(x+\Delta x)-3x}{\Delta x} = \frac{3x+3\Delta x-3x}{\Delta x} = \frac{3\Delta x}{\Delta x} = 3$

   1
   1 1
   1 2 1
   1 3 3 1
   1 4 6 4 1

   b. g(x) = 3x² $\frac{3(x+\Delta x)^2-3x}{\Delta x} = \frac{3(x^2+2x\Delta x+\Delta x^2)-3x}{\Delta x} = \frac{3x^2+6x\Delta x+3\Delta x^2-3x^2}{\Delta x}$
   $= 6x+3\Delta x \to 6x$

   c. h(x) = 3x³ $\frac{3(x+\Delta x)^3-3x^3}{\Delta x} = \frac{3(x^3+3x^2\Delta x+3x\Delta x^2+\Delta x^3)-3x^3}{\Delta x} = \frac{3x^3+9x^2\Delta x+9x\Delta x^2+3\Delta x^3-3x^3}{\Delta x}$
   $= 9x^2+9x\Delta x+3\Delta x^2 \to 9x^2$

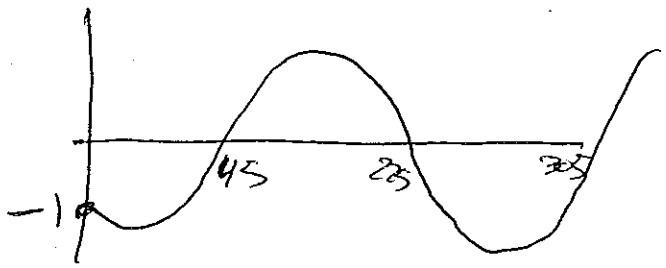
   d. k(x) = 3x⁴ $\frac{3(x+\Delta x)^4-3x^4}{\Delta x} = \frac{3(x^4+4x^3\Delta x+6x^2\Delta x^2+4x\Delta x^3+\Delta x^4)-3x^4}{\Delta x} = 12x^3+18x^2\Delta x+12x\Delta x^2+3\Delta x^3$
   $= 12x^3$

   e. Generalize your findings. What is the IROC of $3x^n$?

   $n \cdot 3x^{n-1}$

2. Let f(x) = sin(x), and g(x) = cos(x). You may use degrees or radians. Please indicate which one you are using.
   a. Sketch a graph of f(x) - g(x). Label all roots, asymptotes, and the y-intercept.

   $\sin x - \cos x = 0$ if, $\sin x = \cos x$
   So, 45° angle & 225°.
   Roots: 45, 225, 308, 485, etc
   No asymptotes
   y-int: $\sin(0) - \cos(0)$
   $0 - 1 = -1$



   b. Sketch a graph of $\frac{g(x)}{f(x)}$ .Label all roots, asymptotes, and the y-intercept.

   $\frac{\cos x}{\sin x}$
   Roots: $\cos x = 0$
   $x = 90, 270, 450$
   Asymptotes: $x = 0, 180, 360$
   y-int: DNE (0 is asymptote)