

Operations with Rational Functions

let $p(x) = \frac{3x^2 - 75}{x^2 + 7x + 10}$ & $q(x) = \frac{x^2 - 3x - 10}{2x^2 - 8}$

$$p(x) = \frac{3(x^2 - 25)}{(x+2)(x+5)} = \frac{3(x+5)(x-5)}{(x+2)(x+5)} = \frac{3(x-5)}{x+2}$$

$$q(x) = \frac{(x-5)(x+2)}{2(x^2 - 4)} = \frac{(x-5)(x+2)}{2(x+2)(x-2)} = \frac{x-5}{2(x-2)}$$

$$p(x) + q(x) = \frac{3(x-5) \cdot 2(x-2)}{(x+2) \cdot 2(x-2)} + \frac{x-5 \cdot (x+2)}{2(x-2) \cdot (x+2)}$$

$$= \frac{6(x-5)(x-2) + (x-5)(x+2)}{2(x+2)(x-2)}$$

$$= \frac{(x-5)(6(x-2) + (x+2))}{2(x+2)(x-2)}$$

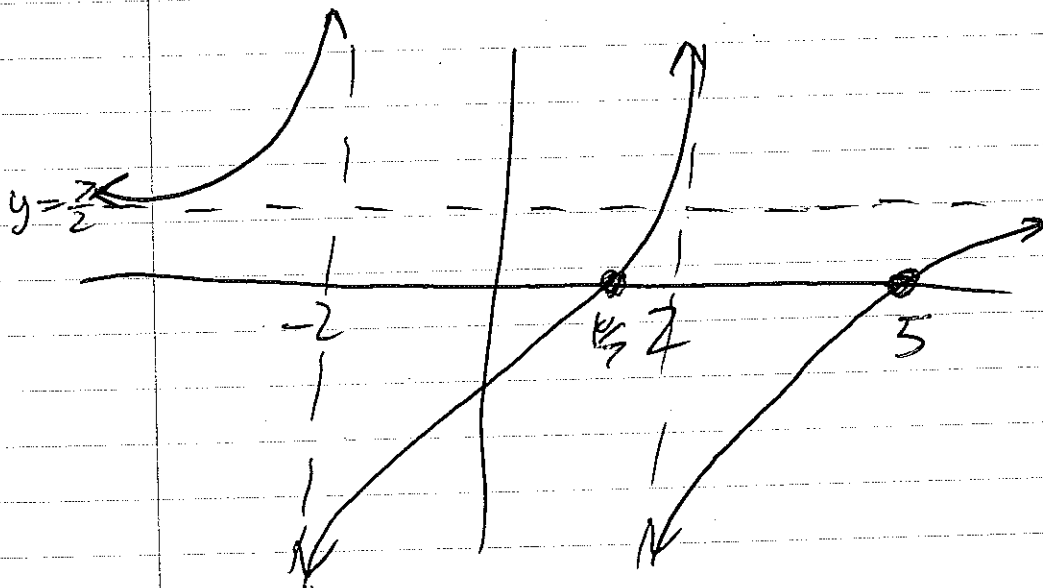
$$= \frac{(x-5)(6x - 12 + x + 2)}{2(x+2)(x-2)}$$

$$= \frac{(x-5)(7x - 10)}{2(x+2)(x-2)}$$

Roots: $x = 5$
 $x = 10/7$

Asymptotes: $x = 2$
 $x = -2$

End Behavior
 $\lim_{x \rightarrow \infty} p(x) + q(x) = 7/2$



$$p(x) - q(x) = \frac{6(x-5)(x-2) - (x-5)(x+2)}{2(x+2)(x-2)}$$

$$= \frac{(x-5)(6(x-2) - (x+2))}{2(x+2)(x-2)}$$

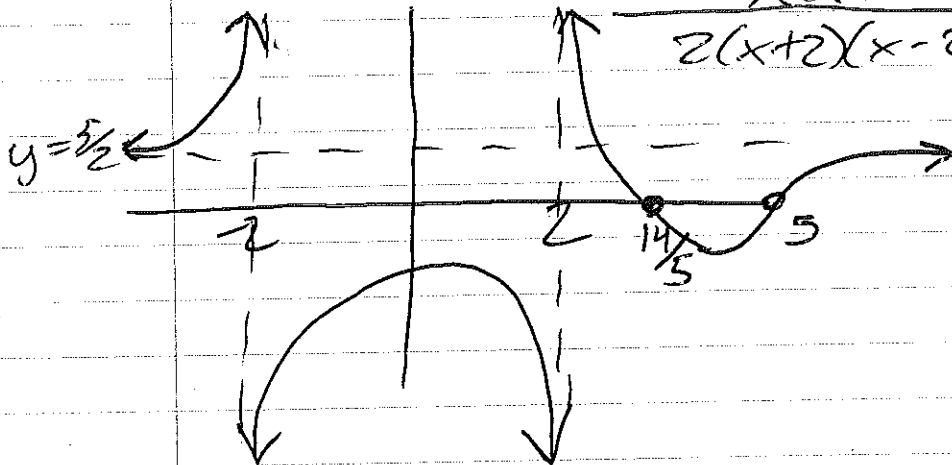
$$= \frac{(x-5)(6x - 12 - x - 2)}{2(x+2)(x-2)}$$

$$= \frac{(x-5)(5x-14)}{2(x+2)(x-2)}$$

Roots: $x=5$
 $x=14/5$

Asymptotes: $x=2$
 $x=-2$

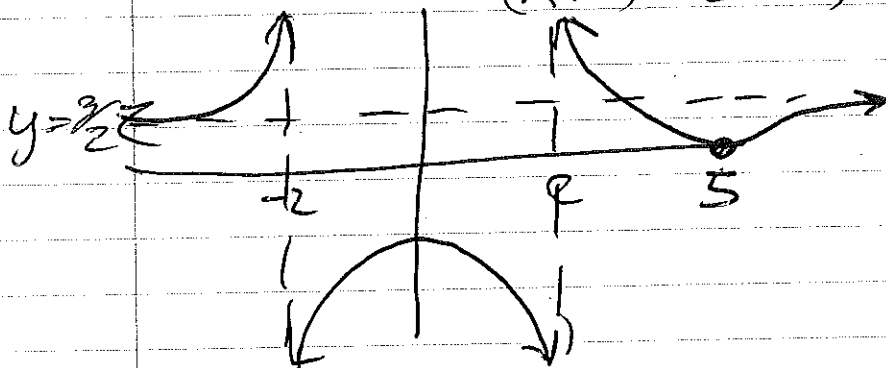
End Behavior:
 $\lim_{x \rightarrow \infty} p(x) - q(x) = 5/2$



$$p(x) \cdot q(x) = \frac{3(x-5)}{(x+2)} \cdot \frac{(x-5)}{2(x-2)} = \frac{3(x-5)^2}{2(x+2)(x-2)}$$

Bounce
Roots: $x=5$
Asymptotes: $x=2$
 $x=-2$

End Behavior:
 $\lim_{x \rightarrow \infty} p(x) \cdot q(x) = 3/2$



$$p(x) \div q(x) = \frac{3(x-5)}{(x+2)} \cdot \frac{2(x-2)}{(x-5)} = \frac{6(x-2)}{(x+2)}$$

Roots: $x=2$
Asymptotes: $x=2$
End Behavior:
 $\lim_{x \rightarrow \infty} p(x) \div q(x) = 6$

