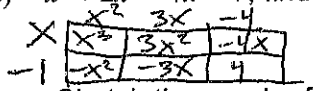
AA6: Polynomials Notes

Questions	Notes
How do you perform mathematical operations with polynomials? • Addition $(x^3 + 3x^2 - 2x + 1) + (-4x^3 - x + x^2 - 3)$ $-3x^3 + 4x^2 - 3x - 2$ • Subtraction $(x^3 + 3x^2 - 2x + 1) - (-4x^3 - x + x^2 - 3)$ $5x^3 + 2x^2 - 1x + 4$ • Multiplication $(x - 2)(x^2 + 5x - 6)$ x^2+5x-6 x | x^3 | $5x^2$ | $-6x$ -2 | $-2x^2$ | $-10x$ | 12 $= x^3+3x^2-16x+12$ • Division $\frac{x^3-4x^2+2x+4}{x-2}$ x^2 $-2x$ -2 x | x^3 | $-2x^2$ | $-2x$ -2 | $-2x^2$ | $4x$ | 4 $= x^2-2x-2$	Add or subtract like terms (Terms with the same exponent) Be careful with positives & negatives. Double negative = positive Multiply or divide using the area model. $L \cdot W = A$. $\frac{A}{W} = L$ W | L | A
How do you rewrite rational expressions? • Addition $\frac{x^2+1}{x-1} + \frac{x^2+2x-1}{x-1} = \frac{2x^2+2x}{x-1}$ • Subtraction $\frac{x^2-1}{x-1} - \frac{x^2-2x-1}{x-1} = \frac{2x}{x-1}$ • Multiplication $\frac{x^2-1}{x+1} \cdot \frac{x+1}{x-1} = \frac{(x+1)(x-1)}{(x+1)} \cdot \frac{x+1}{x-1}$ $= x+1$	Add or subtract needs common denominators. That is what like terms are for fractions. Multiplication does not need a common denominator. Factor & cancel, then multiply the left overs.

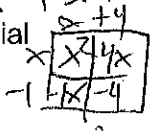
What is the relationship between the Roots and Factors of a Polynomial?

- Use Polynomial Division to find additional roots.

Given that $x = 1$ is one root of the polynomial, $g(x) = x^3 + 2x^2 - 7x + 4$, find the other roots.

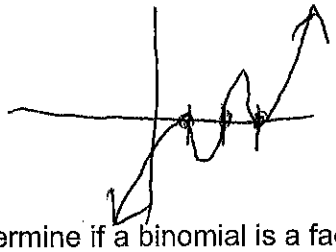


$\rightarrow x^2 + 3x - 4$



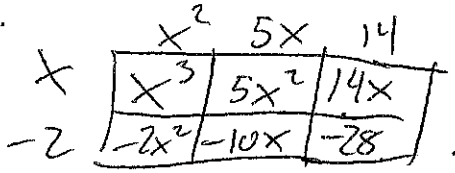
- Sketch the graph of a polynomial

Sketch the graph of $f(x) = (x-1)^2(x-2)(x-3)^2$ showing the roots and end behavior.



- Determine if a binomial is a factor of a polynomial

Is $(x-2)$ a factor of $x^3 + 3x^2 + 4x - 8$? Show how you know.



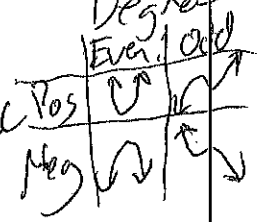
No.

OR

Each root comes from a factor (which may have a multiplicity). If $x=a$ is a root, then $(x-a)$ is a factor. Divide by the factors until you have a quadratic, then factor that.

Steps to graph:

1. Roots = Dots
2. End Behavior
3. Root Behavior



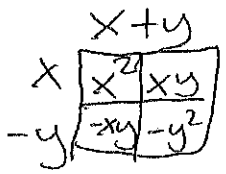
Plug in the root or divide by the factor to tell.

$2^3 + 3(2)^2 + 4(2) - 8 = 20$. $20 \neq 0$

How do you prove identities for Polynomials?

- Difference of two squares

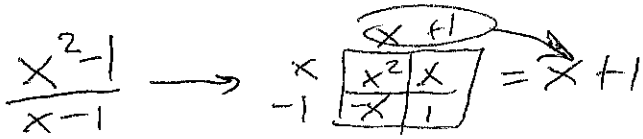
Show that for any values of x and y, $x^2 - y^2 = (x+y)(x-y)$



$= x^2 - y^2$

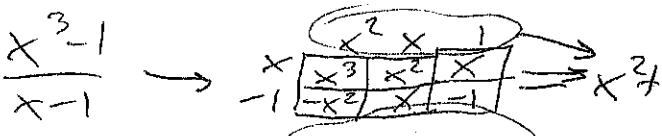
- Generalize patterns of multiplication/division with polynomials.

Find a. $\frac{x^2-1}{x-1}$



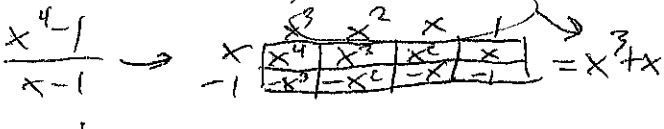
$= x + 1$

b. $\frac{x^3-1}{x-1}$



$= x^2 + x + 1$

c. $\frac{x^4-1}{x-1}$.



$= x^3 + x^2 + x + 1$

Hence, find a general formula for $\frac{x^n-1}{x-1}$ →

$x^{n-1} + x^{n-2} + \cdots + x + 1$