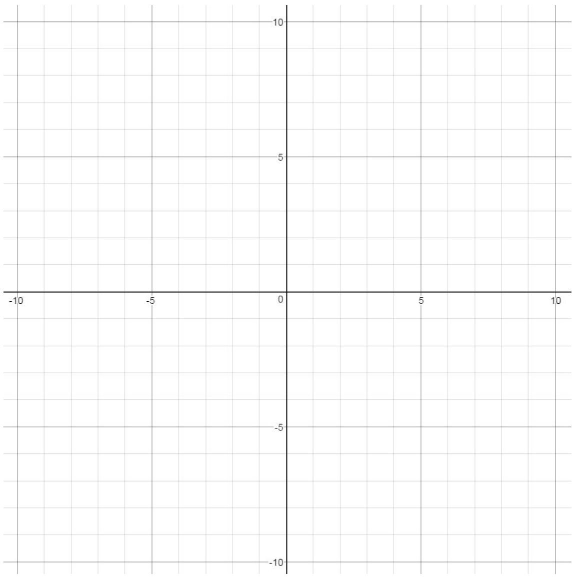
Polynomial Review Packet

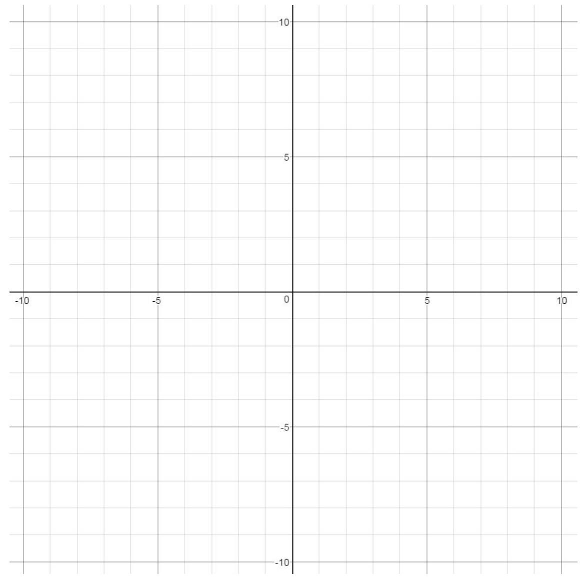
Name:______________________

1. Sketch a graph of each polynomial. Label at least one (x,y) point that is not a root.

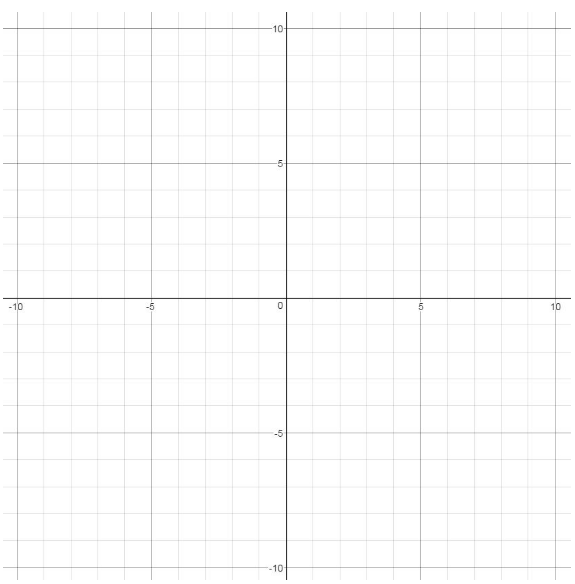
a. $f(x) = -0.2(x - 5)^2(x + 5)^2$



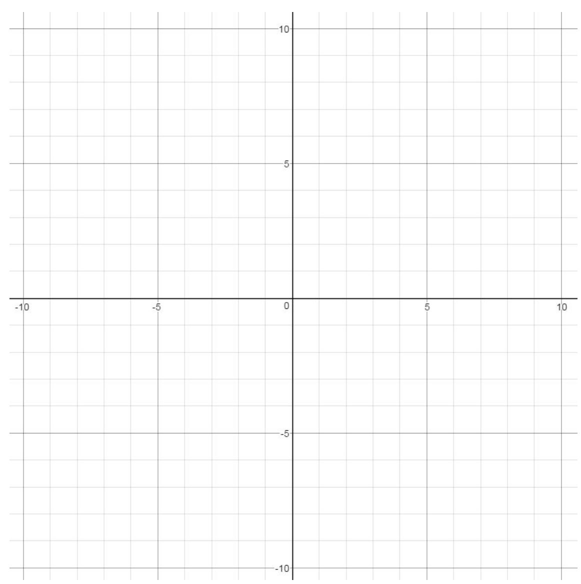
b. $g(x) = \frac{1}{125}(x + 1)(x + 3)^2(x + 5)^3$



c. $h(x) = \frac{1}{32}(x - 2)^3(x + 4)^2$

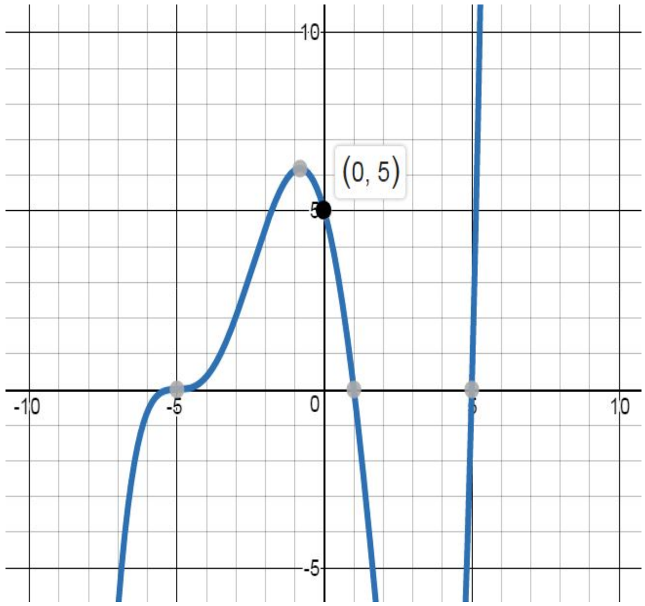


d. $p(x) = \frac{-1}{45}(x + 9)(x + 5)(x - 1)^2$

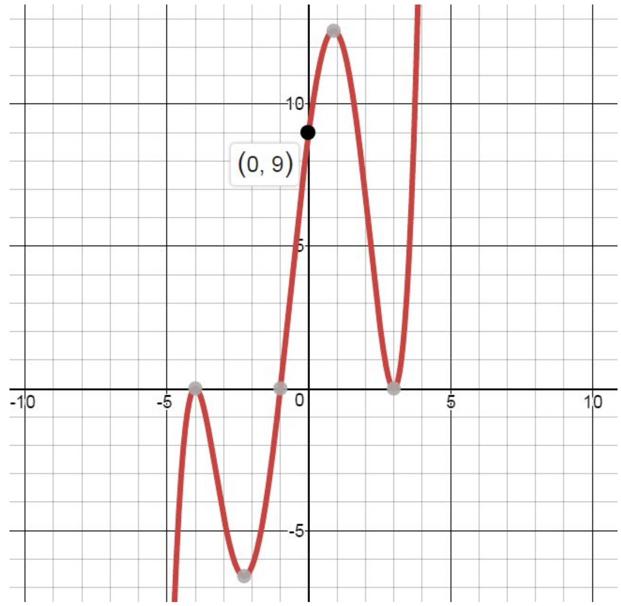


2. Find the exact equation of each polynomial, given a graph.

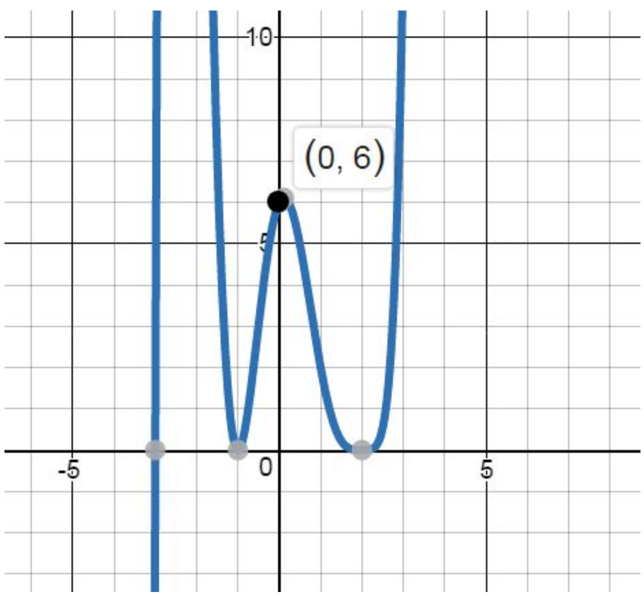
a.



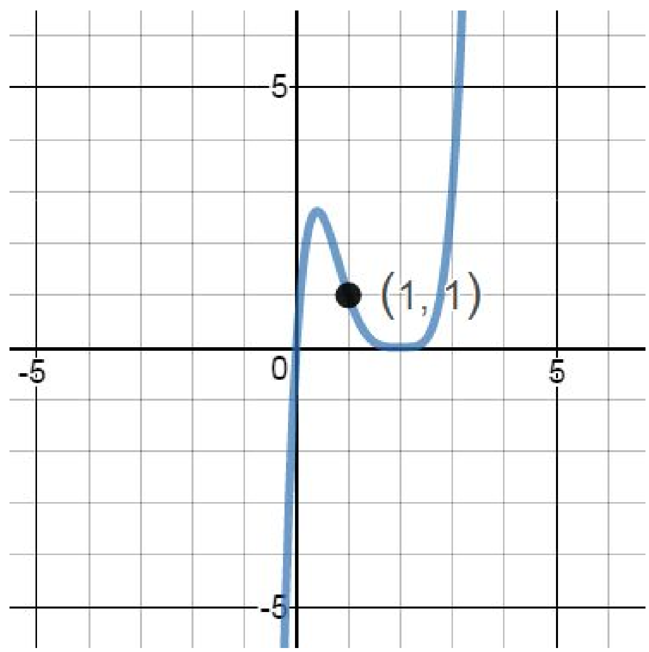
b.



c.



d.



3. Find the exact equation of each polynomial, given a verbal description.

a. Cubic function, real root of 7, complex roots of 1+i, 1-i.

b. Quartic function, complex roots of 3 + i, 3 - i, -2 + i, -2 - i

c. Quintic function (5th degree), real roots of 1, 2, 3, complex roots of 5+ i and its conjugate.

4. Completely factor the polynomial $p(x) = x^4 - 2x^3 - 4x^2 - 8x - 32$, given that two of the roots are 4 and -2

5. Divide the polynomials

a. $\frac{x^4+4x^3-5x^2-36x-36}{x+7}$

b. $\frac{x^3-9x^2+24x-20}{x+9}$

6. Mr. Maurer claims that one of the 3rd roots of 8 is z = $-1+\sqrt{3}\,i$

a. Find |z|.

b. Show by direct computation that $z^3 = 8$

c. Make a geometric argument that $z^3 = 8$, and list the other two 3rd roots of 8. (Hint: We did roots of unity in class, which are on a circle of radius 1. What radius makes sense for $z^3=8$?)

7. You can also use a similar geometric argument to find roots of other complex numbers.

a. Find the four 4th roots of -1.

b. Show by direct computation that $z^4 = -1$ for one of your complex roots.

c. Find the three 3rd roots of $\frac{-\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\,i$

d. Show by direct computation that $z^3 = \frac{-\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\,i$ for one of your complex roots.

8. The function $f(x) = -1x^2 - 16$ does not seem to have any roots. Sketch 3 different view of f(x) to show where the roots are. Make sure to label your axes and any important points.

9.The function $p(x) = x^4 - 81$ has two real roots. The Fundamental Theorem of Algebra states that any polynomial of degree *n* must have exactly *n* roots. Sketch 3 different views of p(x) to show where all *n* roots are. Label your axes and any important points.

10. a. Is it possible for a 4th degree polynomial to have 1 real root? Explain.

b. Is it possible for a 4th degree polynomial to have 2 real roots? Explain.

c. Is it possible for a 4th degree polynomial to have 0 real roots? Explain.

11. The roots of the polynomial f(x) are in an arithmetic sequence where $a_1 = -2$, $d = 4$, $n = 3$. Use the fact that $f(0) = 12$ to find the exact equation of f(x).

12. The roots of the polynomial g(x) are in a geometric sequence where $a_1 = 24$, $m = 0.5$, $n = 4$. Use the fact that $f(0) = 1$ to find the exact equation of f(x).

13. a. Completely factor $p(x) = x^5 - 12x^4 + 25x^3 + 50x^2 + 84x + 392$. Use the Integral Roots Theorem and polynomial division.

b. Sketch a graph of $p(x)$

14. a. Completely factor $q(x) = 2x^4 + 2x^3 + 14x^2 + 18x - 36$. Use the Integral Roots Theorem and polynomial division.

b. Sketch a graph of $q(x)$