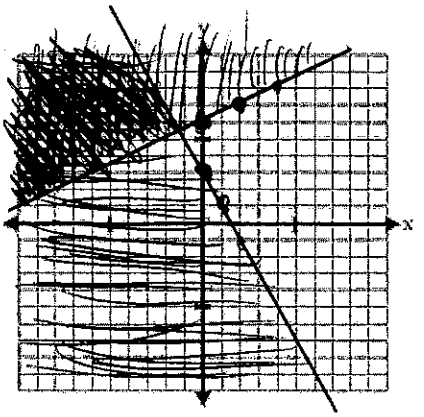
Day 13: Systems of Inequalities

How do you graph a "System of Inequalities"?



Example: Graph: $\begin{cases} y > \frac{1}{2}x + 6 \\ 2x + y \le 3 \end{cases}$

Step 1: Graph the boundary line #1.

$y = \frac{1}{2}x + 6$ (rise / run)

Step 2: Shade inequality #1.

Pick two (2) points: one from either side of the boundary line #1. Substitute each point back into the original inequality #1 and simplify.

Test point 1: (0, 8) True — Above

$8 \stackrel{?}{>} \frac{1}{2}(0) + 6$

$8 > 0 + 6$

$8 > 6$

Test point 2: (0, 2) False — Below

$2 \stackrel{?}{>} \frac{1}{2}(0) + 6$

$2 > 0 + 6$

$2 > 6$

Step 3: Graph the boundary line #2.

$2x + y = 3$

$-2x \quad -2x$

$y = -2x + 3$ (rise / run)

Step 4: Shade inequality #2.

Pick two (2) points: one from either side of the boundary line #2. Substitute each point back into the original inequality #2 and simplify.

$2x + y \le 3$

Test point 1: (4, 2) — Above

$2(4) + 2 \stackrel{?}{\le} 3$

$8 + 2 \le 3$

$10 \le 3$ False

Test point 2: (1, −1) — Below

$2(1) + (-1) \stackrel{?}{\le} 3$

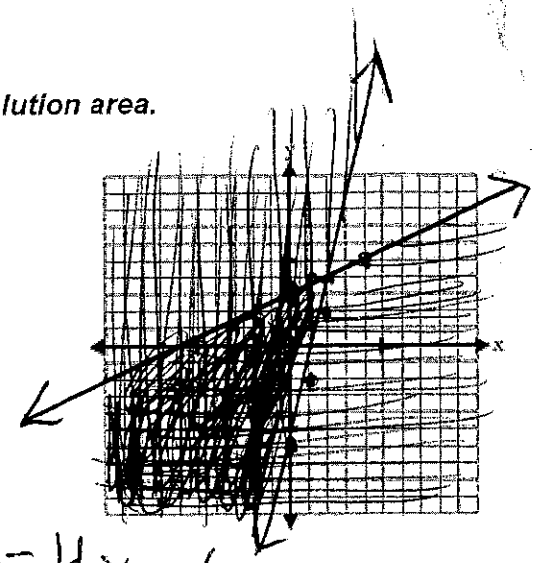
$2 - 1 \le 3$

$1 \le 3$ True

Step 5: Find the region where both inequalities overlap ("double shaded") and darken the area with your pencil. This is the solution area – the region where both inequalities have solutions

Graph the system of inequalities. Show all work and darken the solution area.

1. $\begin{cases} 40x - 10y \le 60 \\ -4x + 8y \le 24 \end{cases}$



Step 1: Graph the boundary line #1.

$40x - 10y = 60$

$-40x \quad -40x$

$\frac{-10y}{-10} = \frac{-40x + 60}{-10} \rightarrow y = \frac{4}{1}x - 6$ ← y-int

Step 2: Test points and shade solution area.

Test point 1: (1, 3) $40x - 10y \le 60$ Test point 2: (6, −6)

Above $40(1) - 10(3) \stackrel{?}{\le} 60$ Below $40(6) - 10(-6) \stackrel{?}{\le} 60$

True $40 - 30$ False $240 + 60$

$10 \le 60$ $300 \le 60$

Step 3: Graph the boundary line #2.

$-4x + 8y = 24$

$+4x \quad +4x$

$\frac{8y}{8} = \frac{4x + 24}{8} \rightarrow y = \frac{1}{2}x + 3$

Step 4: Test points and shade solution area.

Test point 1: (6, 9) $-4x + 8y \le 24$ Test point 2: (−6, −9)

Above $-4(6) + 8(9) \stackrel{?}{\le} 24$ Below $-4(-6) + 8(-9) \stackrel{?}{\le} 24$

False $-24 + 72 \le 24$ True $24 - 72 \le 24$

$48 \le 24$ $-48 \le 24$

Step 5: Find the region where both inequalities overlap ("double shaded") and darken the area with your pencil. This is the solution area – the region where both inequalities have solutions

2. Pick a point in the solution area for both inequalities (your darkened region). Plug the point into each original inequality and test that it is, in fact, a solution to both inequalities.

Systems of Inequalities - Modeling

1) Burgers & Dogs

Jason bought hamburgers and hot dogs for a party. One package of hot dogs cost $4 and one package of hamburgers cost $8.

- Jason can spend **no more than** $48.
- He must buy **at least** 5 packages of hot dogs and hamburgers (total).

If x = *the number of hot dogs* he will buy and y = *the number of hamburgers* he will buy, then we can represent his limitations by:

$4x + 8y \leq 48$
$x + y \geq 5$

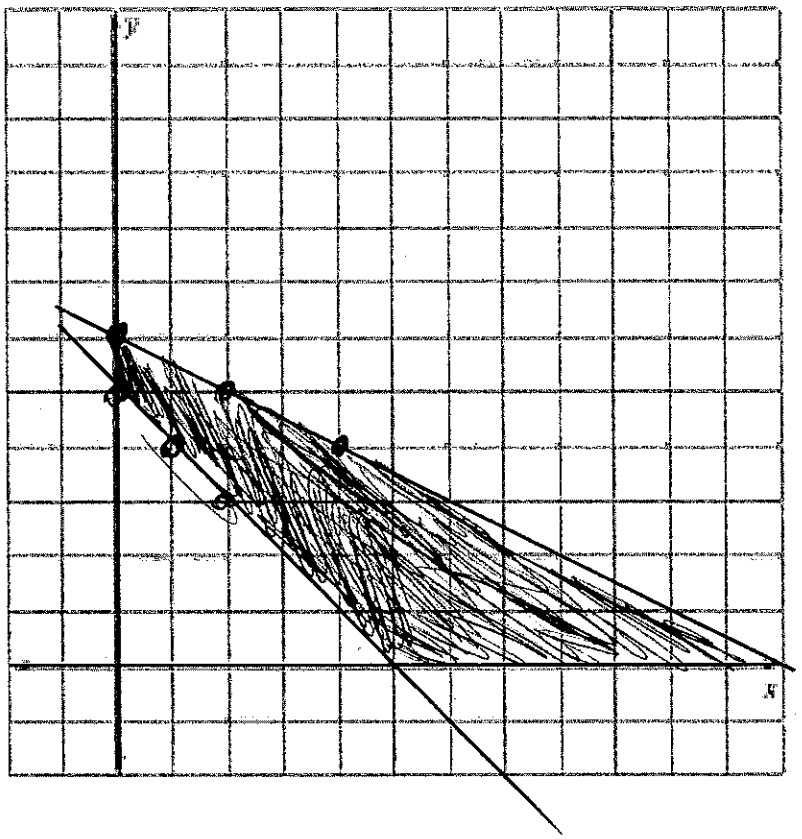
$4x + 8y = 48$
$-4x \quad -4x$

$\frac{8y}{8} = \frac{-4x + 48}{8}$

$y = -\frac{1}{2}x + 6$

$x + y = 5$
$-x \quad -x$

$y = -x + 5$



a) Graph and shade both inequalities on the same grid (above). Darken the solution area.

b) Give four combinations (that fit all four limitations) of hot dogs and hamburgers that Jason can buy.

x = hot dogs, y = hamburgers (0,5) (2,4) (1,5) (2,3)

c) Pick one combination from part b) and explain how it fits Jason's limitation.

(2,3) works because that's 5 packages and $2(4) + 3(8) = 8 + 24 = 32$, which is less than $48

2) Dance Competition

Your school sold tickets to the annual dance competition. The school sold senior citizen tickets for $5 and general admission tickets for $7.

- They needed to raise **at least** $2,100.
- The auditorium, where the competition will take place, **only** has 350 seats.

If x = *the number of senior citizen tickets* and y = *the number of general admission tickets*, then we can represent the limitations by:

$5x + 7y \geq 2100$

$x + y \leq 350$

$5x + 7y = 2100$

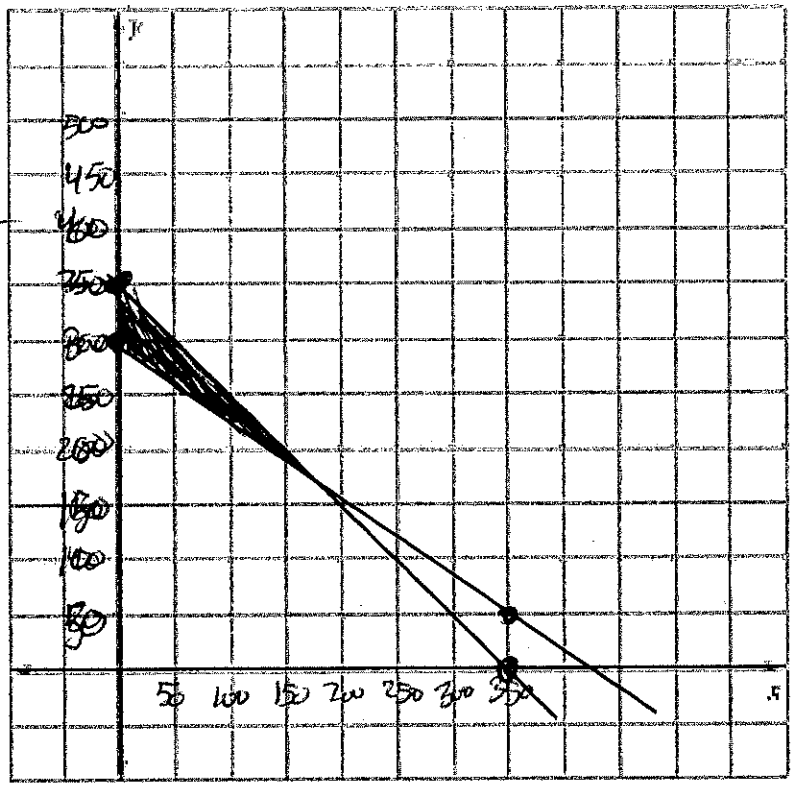
$-5x \quad -5x$

$\frac{7y}{7} = \frac{-5x + 2100}{7}$

$y = -\frac{5}{7}x + 300$

$x + y = 350$

$y = -x + 350$



a) Graph and shade both inequalities on the same grid (above). Darken the solution area.

b) Give four combinations of tickets that the school can sell so they can make the money they want but fit everyone in the auditorium.

(0, 300) (1, 300) (2, 299)

(50, 300)

c) Pick one combination from part b) and explain how it fits the school's limitations.

(2, 299) works because it's less than 350 seats & $2(5) + 299(7) = 10 + 2093 = 2103$, which exceeds 2100.

3) Music Shop

Jake Garcia opens a music shop in which he will sell guitars and basses. There are limitations to his music shop. They are:

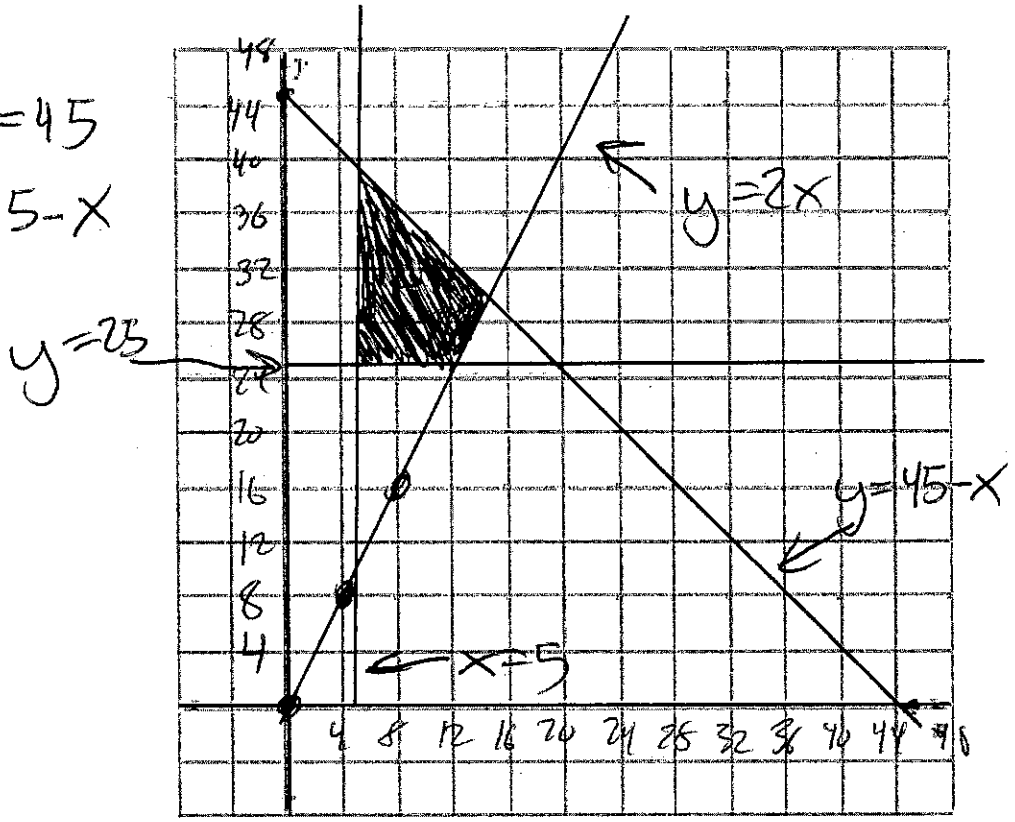
- His store is small, so he can buy **no more than** 45 instruments total.
- Because guitars are more popular than basses, the number of guitars must be **at least** twice the number of basses.
- To get started, Jake must buy **at least** 25 guitars and **at least** 5 basses.

If x = *the number of basses* he will buy and y = *the number of guitars* he will buy, then we can represent his limitations by:

$$\begin{cases} x + y \leq 45 \\ y \geq 2x \\ y \geq 25 \\ x \geq 5 \end{cases}$$

$x+y=45$

$y=45-x$



a) Graph and shade all four inequalities on the same grid (above). Darken the solution area.

b) Give four combinations (that fit all four limitations) of basses and guitars that Jake can buy to start his music shop.

(8,25) (8,26) (8,27) (8,28)

c) Pick one combination from part b) and explain how it fits Jake's limitations for his music shop.

(8,25) works. More than 5 basses, at least 25 guitars, 25 is more than double 8, and 8+25=33, which is less than 45.

4) Saturday Market

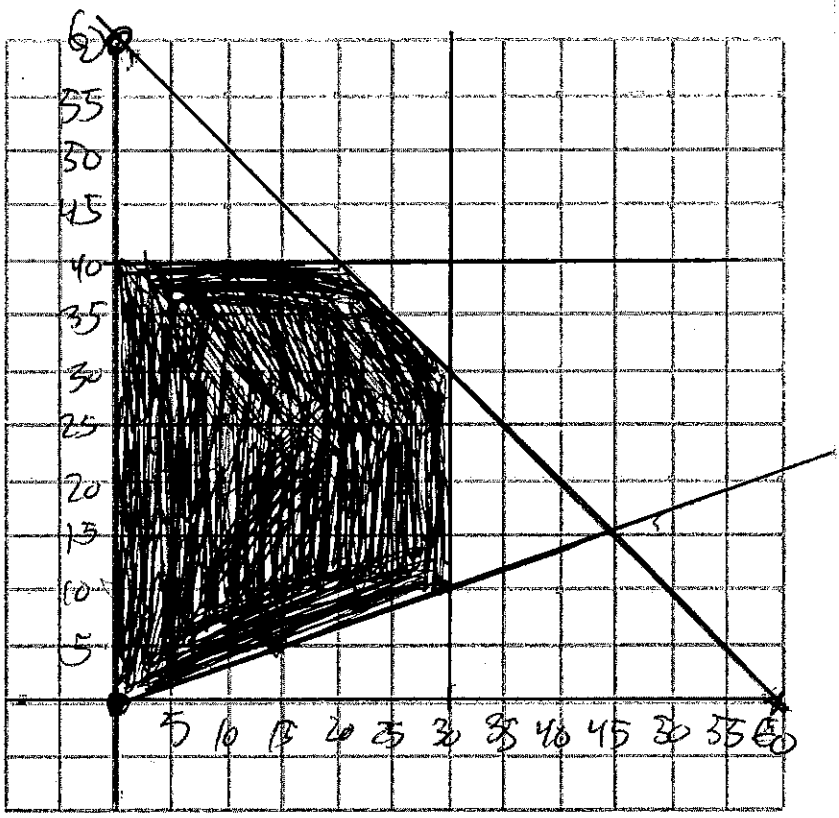
Sandy has decided to open up her own fruit cart at the Portland Saturday Market. To make things easier on her new business, she will only sell apples and pears. She **only** has enough room in her cart for 60 pieces of fruit total and, since she knows people in Portland like apples more than pears, she would like to begin with **at least** three times as many apples as pears. Lastly, she knows she can only receive a **maximum** of 30 apples and 40 pears from her local supplier.

If x = *the number of apples* and y = *the number of pears*, then we can represent her limitations by:

$$\begin{cases} x + y \leq 60 \\ 3y \geq x \\ x \leq 30 \\ y \leq 40 \end{cases}$$

$y = 60 - x$

$y = \frac{1}{3}x$



a) Graph and shade all four inequalities on the same grid (above). Darken the solution area.

b) Give four combinations of apples and pears that Sandy can use to begin her fruit cart. The combinations must fit all four limitations.

(5, 15) (5, 20) (5, 25) (5, 30)

c) Pick one combination from part b) and explain how it fits Sandy's limitations.

(5, 30) because $5 \leq 30$, $30 \leq 40$
$3(30) \geq 5$ & $5 + 30 \leq 60$