

1. Factor the polynomial $f(x) = x^2 + 8x - 20$ and use the factors to find the x-intercepts of the function.

$$\begin{array}{|c|c|} \hline x & +10 \\ \hline x & x^2 & 10x \\ \hline -2 & -2x & -20 \\ \hline \end{array} = (x+10)(x-2)$$

$x = -10, x = 2$

2. Explain why you can't factor $g(x) = x^2 + 8x - 1$. How could you solve the equation $x^2 + 8x - 1 = 0$?

Nothing multiplies to be -1 & adds to be 8.

Use completing the square. $x^2 + 8x - 1 = (x+4)^2 - 17 = 0$

3. The Quadratic Formula ($ax^2 + bx + c = 0, x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$) is a shortcut for a long process of solving Quadratic Equations that CANNOT BE FACTORED (full mathematics of the shortcut). To use the shortcut, you follow three steps:

- Make the equation to be solved in the form $ax^2 + bx + c = 0$ -- it is essential to have the equation = 0.
- Identify the values of a, b and c from the equation (these are the coefficients on the x^2 term, the x term and the constant coefficient).
- Use a calculator to evaluate $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ to determine the solutions.

Use the Quadratic Formula to solve each equation below:

a. $2x^2 + 3x - 7 = 0$
 $a=2, b=3, c=-7$
 $x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)}$
 $= \frac{-3 \pm \sqrt{9 + 56}}{4}$
 $= \frac{-3 \pm \sqrt{65}}{4}$
 $= \frac{-3 \pm 8.06}{4}$

b. $x^2 - 4x - 2 = 0$
 $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)}$
 $= \frac{4 \pm \sqrt{16 + 8}}{2}$
 $= \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 4.89}{2}$
 $= \frac{8.89}{2} = 4.445$ and $-\frac{0.89}{2} = -0.445$

c. $x^2 = 3x - 19$
 $x^2 - 3x + 19 = 0$
 $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(19)}}{2(1)}$
 $= \frac{3 \pm \sqrt{9 - 76}}{2}$
 $= \frac{3 \pm \sqrt{-67}}{2}$

4. All of the above examples, have 2 solutions. Is it possible for a Quadratic Equation to have only 1 solution? Explain why or why not. How could using the Quadratic Formula give you only one solution?

Yes. If $b^2 - 4ac = 0$ then you are taking the square root of 0, which only has one value. Positive #'s give 2 values, Negative #'s give 2 imaginary values.

5. The equation $x^2 + 6x + c = 0$ has only one solution. What must be true about c? How do you know?

$c = 9$ so that $x^2 + 6x + c$ is a perfect square. $(x+3)^2 = x^2 + 6x + 9$.

6. Show that $4x^2 + 4x = -1$ has only one solution.

$$4x^2 + 4x + 1 = 0$$

$$a=4, b=4, c=1$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(4)(1)}}{2(4)} = \frac{-4 \pm \sqrt{16-16}}{8} = \frac{-4 \pm \sqrt{0}}{8} = \frac{-4}{8} = \frac{-1}{2}$$

7. Is it possible that a Quadratic Equation has zero real solutions? Explain why or why not. How could using the Quadratic Formula give you no real solutions?

Yes, if ~~the~~ $b^2 - 4ac < 0$, then you are taking the square root of a negative #. Negative #s don't have real square roots, but they do have imaginary solutions.

8. The equation $x^2 + 6x + c = 0$ has no real solutions. What must be true about c ? Be specific.

If $b^2 - 4ac < 0$, i.e. $6^2 - 4(1)(c) < 0$, then

$$36 - 4c < 0, \text{ so } \frac{36}{4} < \frac{4c}{4} \rightarrow 9 < c$$

AKA, if $c > 9$, there are no real solutions

9. For each Quadratic Equation below, determine whether the equation has 2 real solutions, 1 real solution or no real solutions?

a. $x^2 = 7x - 2$

$$x^2 - 7x + 2 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{7 \pm \sqrt{49-8}}{2}$$

$$\frac{7 \pm \sqrt{41}}{2} \leftarrow 2 \text{ real solutions}$$

b. $-10x^2 + 60x - 90 = 0$

$$x = \frac{-60 \pm \sqrt{60^2 - 4(-10)(-90)}}{2(-10)}$$

$$= \frac{-60 \pm \sqrt{3600 - 3600}}{-20}$$

$$= \frac{-60 \pm \sqrt{0}}{-20} \uparrow$$

1 real solution

c. $0.25x^2 = 3.11x - 18.2$

$$0.25x^2 - 3.11x + 18.2 = 0$$

$$x = \frac{-(-3.11) \pm \sqrt{(-3.11)^2 - 4(0.25)(18.2)}}{2(0.25)}$$

$$= \frac{3.11 \pm \sqrt{9.6721 - 18.2}}{0.5}$$

$$= \frac{3.11 \pm \sqrt{-8.52}}{0.5} \uparrow$$

2 imaginary solutions

10. Show that the quadratic function $f(x) = x^2 + 1$ has NO REAL ROOTS.

$$a=1, b=0, c=1$$

$$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{\pm \sqrt{-4}}{2} \leftarrow 2 \text{ imaginary solutions}$$