

Rational Expressions Worksheet

Reducing Rational Expressions

$$1. \text{ Reduce } \frac{x^2-9}{x-3} = \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}} = x+3$$

$$2. \text{ Reduce } \frac{y^2-5y-6}{y^2-1} = \frac{(y-6)\cancel{(y+1)}}{\cancel{(y+1)}(y-1)} = \frac{y-6}{y-1}$$

$$3. \text{ Reduce } \frac{x^2-3x+ax-3a}{x^2-ax-3x+3a} = \frac{x(x-3)+a(x-3)}{x(x-a)-3(x-a)} = \frac{\cancel{(x-3)}(x+a)}{\cancel{(x-3)}(x-a)} = \frac{x+a}{x-a}$$

$$4. \text{ Reduce } \frac{a-b}{b-a} = \frac{a-b}{-(a-b)} = \frac{1}{-1} = -1$$

$$5. \text{ Reduce } \frac{x^2-25}{5-x} = \frac{(x+5)\cancel{(x-5)}}{-1\cancel{(x-5)}} = -(x+5) = -x-5$$

$$6. \text{ Reduce } \frac{2x^3+2x^2-24x}{x^3+2x^2-8x} = \frac{2x\cancel{(x^2+x-12)}}{x\cancel{(x^2+2x-8)}} = \frac{2\cancel{(x+4)}(x-3)}{\cancel{(x+4)}(x-2)} = \frac{2x-6}{x-2}$$

Multiplying and Dividing Expressions

$$1. \text{ Multiply } \frac{\cancel{2}x^3}{\cancel{8}x^3} \cdot \frac{\cancel{9}y^3}{\cancel{12}x^2} = \frac{2x^3y^3}{9y^3x^2} = \frac{2x}{9y}$$

$$2. \text{ Multiply } \frac{x-3}{x^2-4} \cdot \frac{x+2}{x^2-6x+9} = \frac{\cancel{x-3}}{(\cancel{x-2})(x-2)} \cdot \frac{\cancel{x+2}}{\cancel{(x-3)}(x-3)} = \frac{1}{(x-2)(x-3)}$$

2

3. Divide $\frac{3x-9}{x^2-x-20} \cdot \frac{x^2+2x-15}{x^2-25} = \frac{3(x-3)(x+5)(x-3)}{(x-5)(x+4)(x-5)(x+5)} = \frac{3(x-3)^2}{(x-5)^2(x+4)}$

As a division problem, multiply by reciprocal

4. Multiply $(x^2-49) \left(\frac{x+4}{x+7} \right)$

$$\frac{3(x-3)}{(x-5)(x+4)} \cdot \frac{(x+5)(x+3)}{(x+5)(x+3)} = \frac{3}{x+4}$$

$$(x-7)(x+7)(x+4) = (x-7)(x+4) = x^2 - 3x - 28$$

5. Multiply $a(a+5)(a-5) \left(\frac{a+4}{a^2-5a} \right) = a(a+5)(a-5) \frac{(a+4)}{a(a-5)} = a^2 + 9a + 20$

Adding and Subtracting Expressions *Common Denominator*

1. Add $\frac{-2}{x^2-2x-3} + \frac{3}{x^2-9} = \frac{-2}{(x-3)(x+1)} + \frac{3}{(x+3)(x-3)} = \frac{-2(x+3) + 3(x+1)}{(x-3)(x+1)(x+3)}$

$$= \frac{-2x-6+3x+3}{(x-3)(x+1)(x+3)} = \frac{x-3}{(x-3)(x+1)(x+3)} = \frac{1}{x^2+4x+3}$$

$$= \frac{x+1}{x^2-1} = \frac{x+1}{(x+1)(x-1)} = \frac{1}{x-1}$$

3. Subtract $\frac{x+4}{2x+10} - \frac{5}{x^2-25}$

$$\frac{(x+5)(x+4)}{(x-5)2(x+5)} - \frac{5}{(x+5)(x-5)} \left(\frac{2}{2} \right) = \frac{(x-5)(x+4) - 10}{2(x-5)(x+5)} = \frac{x^2 - x - 20 - 10}{2(x-5)(x+5)}$$

4. Add $\frac{5}{x} + \frac{3}{x}$

$$= \frac{8}{x}$$

$$\frac{x^2 - x - 30}{2(x-5)(x+5)} = \frac{(x-6)(x+5)}{2(x-5)(x+5)} = \frac{(x-6)}{2(x-5)}$$

Complex Expressions

1. Simplify $\frac{3}{4} \cdot \frac{8}{5} = \frac{3 \cdot 8}{4 \cdot 5} = \frac{3 \cdot 4 \cdot 2}{1 \cdot 5} = \frac{6}{5}$

Rational Equations

1. Solve for a: $\frac{6}{a-4} = \frac{3}{8} \rightarrow 3(a-4) = 6 \cdot 8$
 $3a - 12 = 48$
 $3a = 60$
 $\frac{3a}{3} = \frac{60}{3}$

$a = 20$

2. Solve for x: $\left(\frac{x}{x-2} + \frac{2}{3} = \frac{2}{x-2}\right) \cdot (x-2)$

$3 \cdot \left(x + \frac{2}{3}(x-2) = 2\right) \cdot 3$

$3x + 2(x-2) = 6 \rightarrow$

$3x + 2x - 4 = 6$

$5x = 10$

$x = 2$. But $x \neq 2$. So
 no solution.

3. Solve for x: $\frac{5}{x^2-3x+2} - \frac{1}{x-2} = \frac{1}{3x-3}$

$(x-2) \left[\frac{5}{(x-2)(x-1)} - \frac{1}{x-2} = \frac{1}{3(x-1)} \right] \cdot (x-2)$

$x-1 \left[\frac{5}{x-1} - 1 = \frac{x-2}{3(x-1)} \right] \cdot x-1$

$3 \cdot \left[5 - (x-1) = \frac{x-2}{3} \right] \cdot 3$

$15 - 3(x-1) = x-2$

$15 - 3x + 3 = x-2$

$12 - 3x = x-2$

$14 = 4x$

$\frac{14}{4} = x$

4. Solve for x: $3 + \frac{1}{x} = \frac{10}{x^2}$

$x^2 \left[3 + \frac{1}{x} = \frac{10}{x^2} \right] \cdot x^2$

$3x^2 + x - 10 = 0$

$3x^2 + x = 10 \rightarrow$

$(3x-5)(x+2)$

$x = \frac{5}{3} \quad x = -2$

5. Solve for y: $x = \frac{y-4}{y-2}$

$x(y-2) = (y-4)$

$xy - 2x = y - 4$
 $+2x$ $-y$ $+2x$ -4

$xy - y = 2x - 4$

$y(x-1) = 2x-4$

$y = \frac{2x-4}{x-1}$

$$2. \text{ Simplify } \frac{\frac{x-2}{x^2-9}}{\frac{x^2-4}{x+3}} = \frac{x-2}{x^2-9} \cdot \frac{x+3}{x^2-4} = \frac{\cancel{x-2}}{(x+3)(x-3)} \cdot \frac{\cancel{x+3}}{(x+2)(x-2)}$$

$$= \frac{1}{(x-3)(x+2)} = \frac{1}{x^2-x-6}$$

$$3. \text{ Simplify } 2 - \frac{3}{x + \frac{1}{3}} = 2 - \frac{3 \cdot 3}{(x + \frac{1}{3}) \cdot 3} = \frac{2(3x+1) - 9}{3x+1} = \frac{6x+2-9}{3x+1}$$

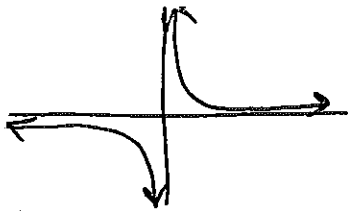
$$= \frac{6x-7}{3x+1}$$

$$4. \text{ Simplify } \frac{(x + \frac{1}{y}) \cdot y \cdot x}{(y + \frac{1}{x}) \cdot y \cdot x} = \frac{x^2 y + x}{y^2 x + y} = \frac{x(xy+1)}{y(xy+1)} = \frac{x}{y}$$

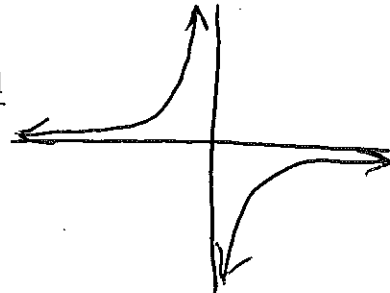
Graphing Rational Functions

1. Graph each equation.

a. $y = \frac{1}{x}$

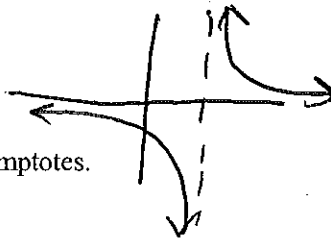


b. $y = \frac{-1}{x}$

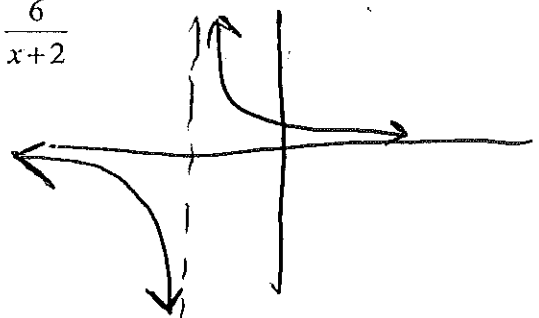


2. Graph each equation.

a. $y = \frac{6}{x-2}$



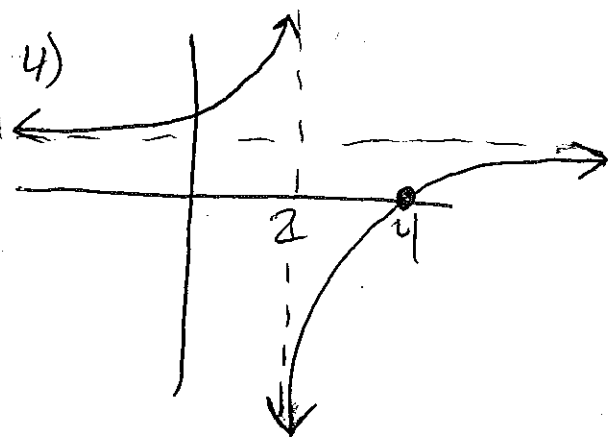
b. $y = \frac{6}{x+2}$



3. Examine horizontal asymptotes.

4. Graph $y = \frac{x-4}{x-2}$

root = 4
Asymptote = 2
E.g. $\lim_{x \rightarrow \infty} y = \frac{1}{1} = 1$



5. Graph $y = \frac{3x-1}{2x+1}$

root = 1/3
Asymptote = -1/2
E.g. $\lim_{x \rightarrow \infty} y = \frac{3}{2} = 1.5$

