Roots, Multiplicity, & End Behavior.

Graphing polynomials depends on 3 things: roots, multiplicity, & end behavior. I'll break each of those down & then give some examples of graphing polynomials.

Roots: x values that make your polynomial equal zero. AKA x-intercepts or zeroes.

Formally, $x=r$ is a root of $f(x)$ if $f(r)=0$.

Examples: $f(x)=(x-4)^3(x+3)^2$ has roots of $x=4$ & $x=-3$

because $f(4)=(4-4)^3(4+3)^2=(0)^3(7)^2=0$ &

$f(-3)=(-3-4)^3(-3+3)^2=(-7)(0)=0.$

$g(x)=-17x^2(x-\pi)(x+e)^3(x-\sqrt{2})$ has roots of $x=0$, $x=\pi$, $x=-e$, $x=\sqrt{2}$

because $g(0)=g(\pi)=g(-e)=g(\sqrt{2})=0.$

Multiplicity: When a factor is raised to a power, we say the corresponding root has a multiplicity. This changes the behavior of the polynomial at that root.

Examples, $f(x) = (x-4)^3(x+3)^2$

The root $x=4$ has multiplicity 3.

The root $x=-3$ has multiplicity 2.

Roots with multiplicity of 1 look linear / or \

Roots with multiplicity of 2 look parabolic $\cup$ or $\cap$

Roots with multiplicity of 3 look cubic or

For any multiplicity bigger than 3, just see if it's even or odd & make it look parabolic or cubic, respectively.

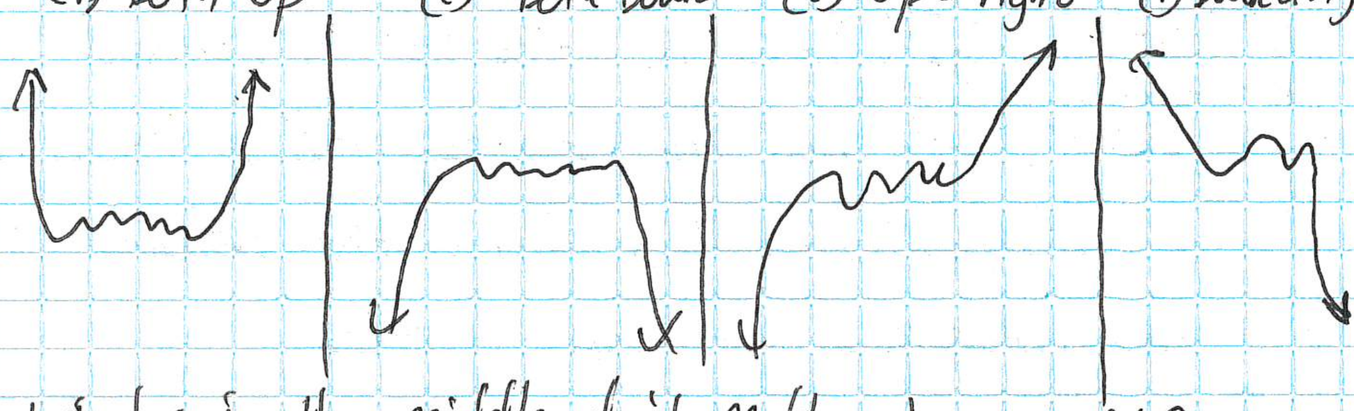
$\boxed{\text{End Behavior:}}$ End behavior describes the behavior of your polynomial as $x \to \infty$ or $x \to -\infty$.

Recall that $x \to \infty$ is the same as "Moving right"

and that $x \to -\infty$ is the same as "Moving left"

There are only 4 possibilities:

(1) Both up (2) Both Down (3) Up & right (4) Down & right



The wiggles in the middle don't matter because we are focusing on <u>end</u> behavior.

To calculate end behavior, look at the degree & leading coefficient (LC).

Degree / L.C.	Even	Odd
Positive	Both up	~~Up & right~~
Negative	Both down	Down & right

← Notice positive L.C.s end up

← Notice negative L.C.s end down

Notice even degrees are roughly symmetrical

Notice odd degrees are anti-symmetrical

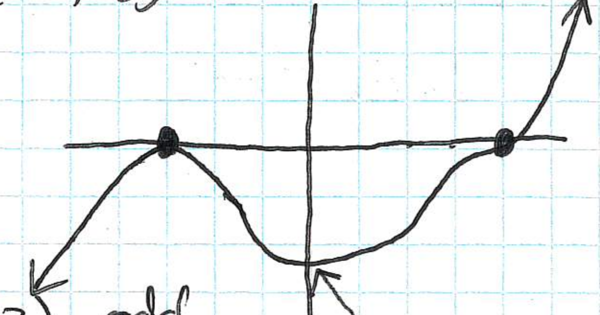
Examples: 1) $f(x) = (x-4)^3(x+3)^2$

1) Roots: $x=4$ $\quad x=-3$

2) Multiplicities: 3 $\quad$ 2

3) End Behavior: Degree $= 5$ $(3+2)$ odd

L.C. $= 1$ positive



Bonus: y-int $= (0-4)^3(0+3)^2 = -64 \cdot 9 = -576$

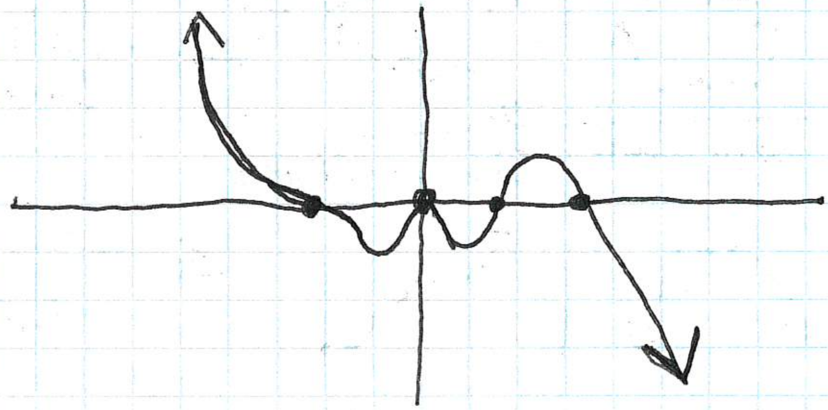
2) $g(x) = -17x^2(x-\pi)(x+e)^3(x-\sqrt{2})$

1) Roots: $x=\pi$, $x=0$, $x=-e$, $x=\sqrt{2}$

2) Multiplicity: 1 2 3 1

3) E. B.: Degree = 7 $(2+1+3+1)$ odd

L. C. = -17 negative



3) $h(x) = -2(x-5)^3(x-3)^2(x-1)(x+1)^2(x+3)^3x$

1) Roots: $x=5$, $x=3$, $x=1$, $x=-1$, $x=-3$, $x=0$

2) Multiplicity: 3 2 1 2 3 1

3) E. B.: Degree = 12 $(3+2+1+2+3+1)$ even

L. C. = -2 negative