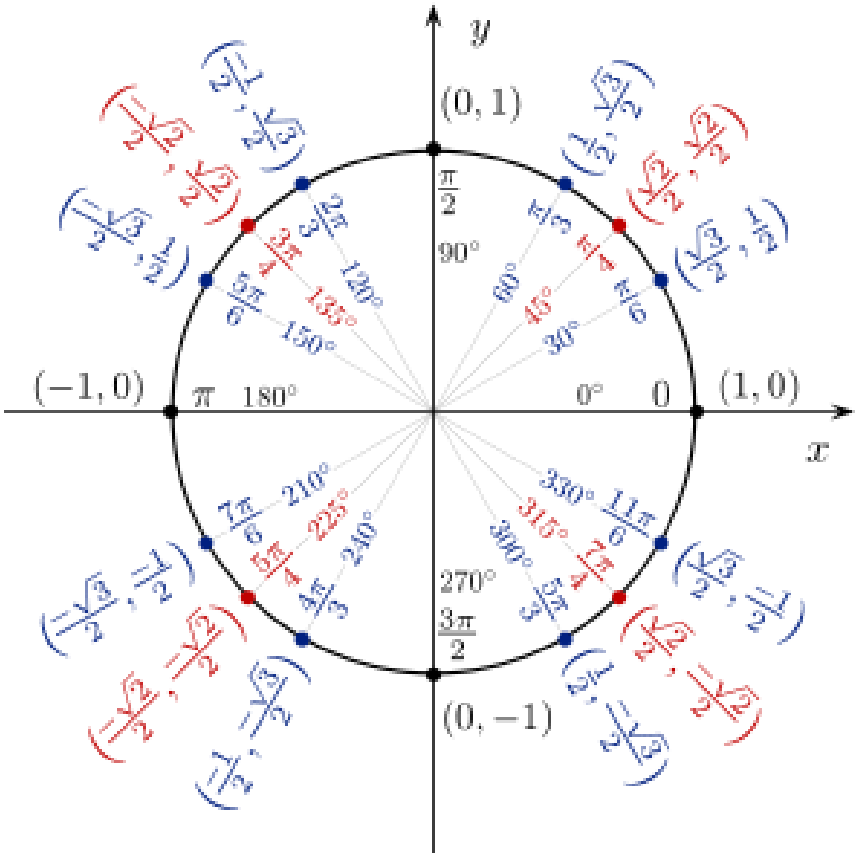
Roots of Trig Functions

The Unit Circle is very helpful for understanding trigonometric functions, like sinx and cosx.



Recall that an x-coordinates of the unit circle is the cosine of the central angle, and the y-coordinate of the unit circle is the sine of the central angle. ($\cos\theta$ =x, $\sin\theta$ =y)

When you graph the functions cosx or sinx, what you have done is made the x-axis turn into the θ axis. You are unwrapping the circle, and placing its angles along the x-axis.

Recall also that a root of a function is a value that you plug in to get an answer of zero.

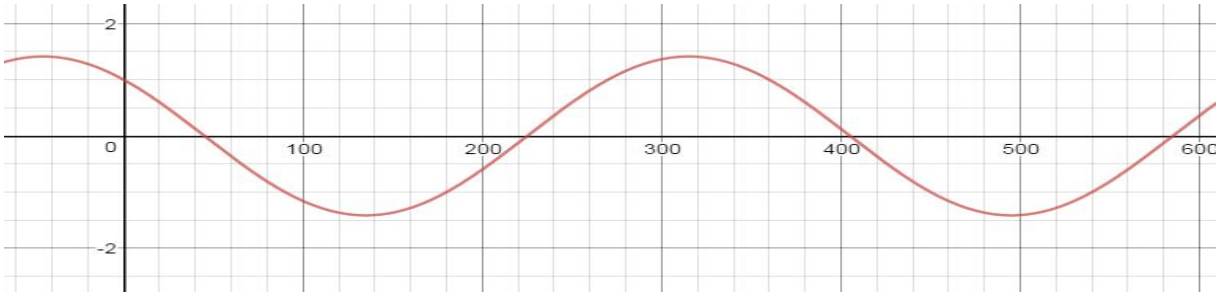
So the roots of cosx are the angles with an x value of zero. Here are a few of them: 90, 270, 450, 630, etc. You can also use radians to measure the angles, so you would get: $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{5\pi}{2}$, $\frac{7\pi}{2}$

Similarly, the roots of sinx are the angles with a y value of zero. Here are the first few: 0, 180, 360, 540, etc. Remember that you can rotate more than a full circle, or you could rotate negatively. Here are the first few negatives: 0, -180, -360, etc.

Finally, if you add two functions and get 0, then one must be the opposite of the other. If you subtract two functions and get 0, they must be equal.

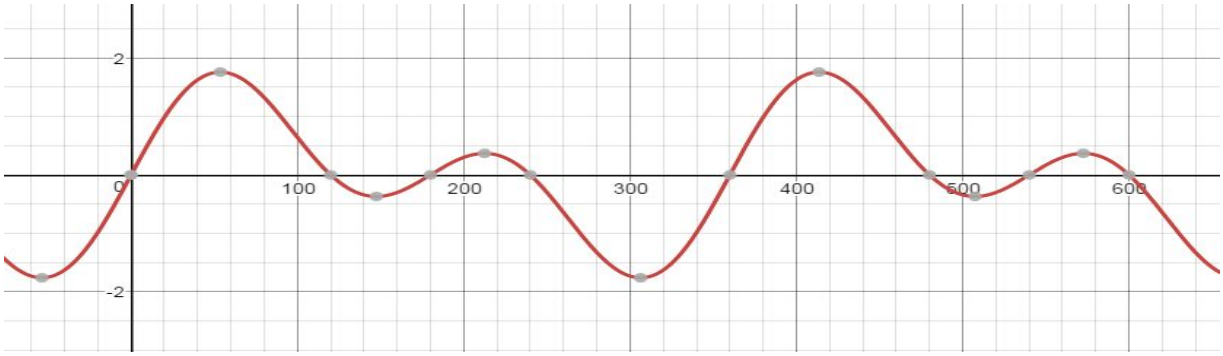
Examples: 1. f(x) = cos(x) - sin(x)

The roots of cosx - sinx are the angles where cosine and sine are equal. Using the unit circle, I see that at 45, and 225 both sine and cosine are $\frac{\sqrt{2}}{2}$. I also can rotate more than a full circle, so the next two roots are 405 and 585. The only other thing is to figure out if the graph is above or below the x axis. When I plug in 0, I get cos(0) - sin(0) = 1 - 0 = 1. So the graph starts above.



2. g(x) = sin(x) + sin(2x)

The roots of sin(x) + sin(2x) are the angles where sin(x) is the opposite of sin(2x). So I look on the unit circle for where the y-coordinates are equal, but opposite, and where one angle is twice as large as the other. Remember also that - 0 = 0 and 2(0) = 0. I see that both equal 0 at angle 0. I also see that sin(120) = $\frac{\sqrt{3}}{2}$ and sin(240) = $\frac{-\sqrt{3}}{2}$. The next angle is 180, because sin(180)=sin(360)=0. The next angle is 240, because sin(240) = $\frac{-\sqrt{3}}{2}$, and sin(480) = $\frac{\sqrt{3}}{2}$.



Exercises:

1. f(x) = cos(x) + cos(2x)
2. g(x) = sin(x) - sin(2x)
3. h(x) = sin(x) + sin(3x)
4. k(x) = sin(x) - sin(3x)